

## Moving Cluster Method:

The angle between a cluster star and the convergent point is angle "a". You already know the relationship of distance, proper motion, and tangential velocity. It is:

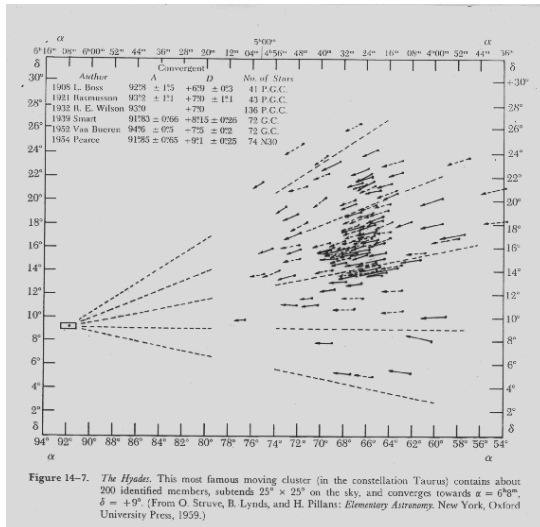
$$V_r = V_s \cos (a)$$

$$\text{and } V_t = V_s \sin (a)$$

$$\text{so } V_t = V_r \tan (a)$$

$$\text{Since } V_t = 4.74 \mu \text{ [arcsec/yr] } d \text{ [pc]}$$

$$d \text{ [pc]} = V_r \tan (a) / 4.74 \mu$$

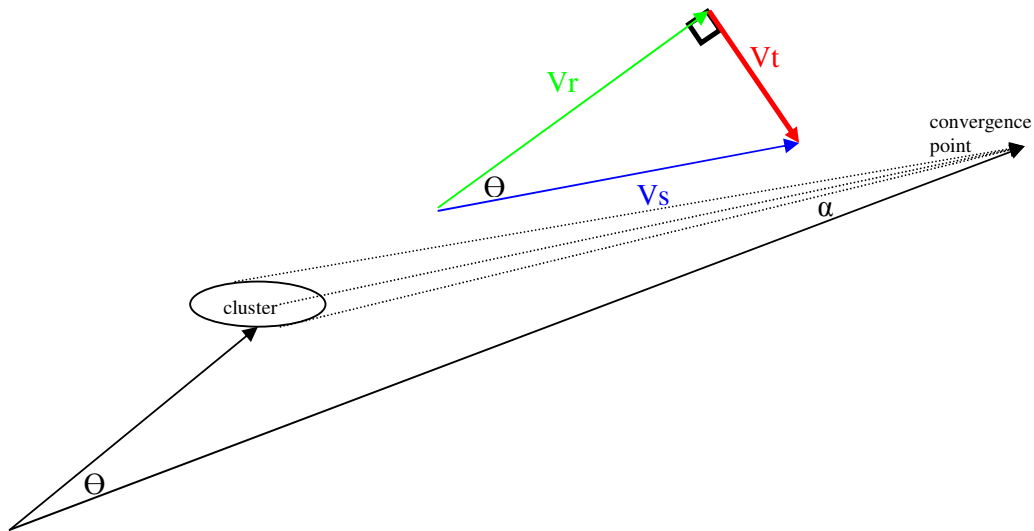


$$X_{[AU]Tangential} = V_t \left[ \frac{AU}{year} \right] \times year = \theta \times d_{[pc]} \Rightarrow \frac{\theta_{[arcsec]}}{year} \times \left[ \frac{1AU}{1arc\ sec} \right] = V_t \left[ \frac{AU}{year} \right]$$

$$\frac{AU}{year} = \frac{1.5 \times 10^8 km}{365 \times 24 \times 3600_{sec}} = 4.74 \frac{km}{sec}$$

$$d_{[pc]} = \frac{V_t \left[ \frac{AU}{year} \right]}{\left[ \frac{\theta}{year} \right]} = \frac{V_r \left[ \frac{km}{sec} \right]}{4.74 \times \left[ \frac{\theta}{year} \right]} = \frac{V_r \left[ \frac{km}{sec} \right] \tan \theta}{4.74 \times \left[ \frac{\theta}{year} \right]}$$

Where  $\mu$  is the proper motion in arc-seconds/year.



### Example:

The angle between a cluster star and the convergent point is angle  $28.5^\circ$ .  
 The radial velocity is  $30\text{km/sec}$ .  
 and the proper motion is  $= 2.85\text{arcsec/year}$

You already know the relationship of distance, proper motion, and tangential velocity.  
 It is:

$$V_r = V_s \cos(a)$$

$$\text{and } V_t = V_s \sin(a)$$

$$\text{so } V_t = V_r \tan(a)$$

$$\text{Since } V_t = 4.74 \mu [\text{arcsec/yr}] d [\text{pc}]$$

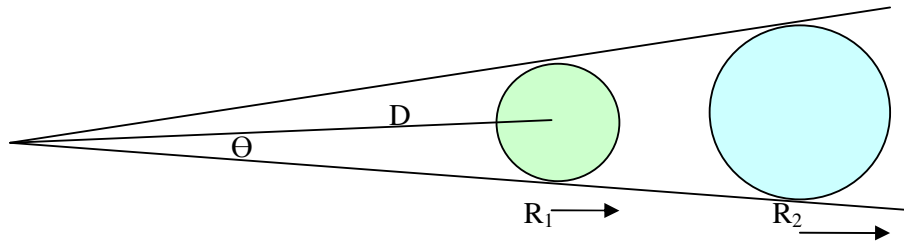
$$d [\text{pc}] = V_r \tan(a) / 4.74 \mu$$

hence:

$$d [\text{pc}] = 30\text{km/sec} * \tan(28.5^\circ) / (4.74 * 2.85\text{arcsec/year}) \approx 1.2 \text{ pc}$$

### Approaching Cluster

When a cluster of stars approaches us directly we can only measure radial velocity through red or blue shift, and its apparent radius. We assume that there is not much of a transverse velocity, so the change in its radius is due only to its approaching.



Measuring the angle  $\theta$  we get the relation  $\sin \theta = \frac{R}{d} \approx \theta \Rightarrow R = \theta d$

and  $V_r = \frac{dD}{dt}$ .

The ln of this expression will also give a correct relation so we get:

$$\ln R = \ln \theta + \ln d$$

Since we assume the actual radius does not change in time, the time derivative of the above expression should equal:

$$\frac{1}{R} \frac{dR}{dt} = 0 = \frac{1}{\theta} \frac{d\theta}{dt} + \frac{1}{D} \frac{dD}{dt} \Rightarrow \frac{1}{D} = \frac{1}{\theta} \frac{d\theta}{dt} \cdot \frac{1}{V_r}$$

The angle between a cluster star and the convergent point is angle  $50^\circ$ .

The radial velocity is 30km/sec.

and the proper motion is = 5arcsec/year

You already know the relationship of distance, proper motion, and tangential velocity.

It is:

$$V_r = V_s \cos (a)$$

$$\text{and } V_t = V_s \sin (a)$$

$$\text{so } V_t = V_r \tan (a)$$

$$\text{Since } V_t = 4.74 \mu [\text{arcsec/yr}] d [\text{pc}]$$

$$d [\text{pc}] = V_r \tan (a) / 4.74 \mu$$

hence:

$$d [\text{pc}] = 30\text{km/sec} * \tan (50^\circ) / (4.74 * 5\text{arcsec/year}) \approx 1.5 \text{ pc} \approx 5\text{ly}$$