

$$\begin{aligned}f(x) &= \cos(2x) \\f(x)' &= -2\sin(2x) \\f(x)'' &= -4\cos(2x) \\f(x)''' &= 8\sin(2x)\end{aligned}$$

$$\begin{aligned}f(x) &= \sin(x) \\f(x)' &= \cos(x) \\f(x)'' &= -\sin(x) \\f(x)''' &= -\cos(x)\end{aligned}$$

$$\begin{aligned}f(x) &= \frac{1-x^3}{2+x^2} \\f(x)' &= \frac{-3x^2}{2+x^2} - \frac{1-x^3}{(2+x^2)^2} 2x\end{aligned}$$

$$\begin{aligned}f(x) &= e^{4x} \\f(x)' &= 4e^{4x}\end{aligned}$$

$$1) f(x) = (2x+1)^{1/3}$$

$$f'(x) = (2x+1)^{-2/3} \cdot 2$$

$$f''(x) = (2x+1)^{-5/3} \cdot 4$$

$$2) f(x) = \log(a^x \cdot b^{-x}) = \log(a^x) + \log(b^{-x}) = x \log a - x \log b$$

$$f'(x) = \log a - \log b = \log \frac{a}{b}$$

$$f''(x) = 0$$

$$3) f(x) = \frac{1}{\log x} = (\log(x))^{-1}$$

$$f'(x) = -(\log(x))^{-2} \cdot \frac{1}{x} = -\frac{1}{x(\log x)^2} = -x^{-1}(\log x)^{-2}$$

$$f''(x) = x^{-2}(\log x)^{-2} + 2x^{-1}(\log x)^{-3} \cdot x^{-1} = \frac{1}{x^2(\log x)^2} + \frac{2}{x^2(\log x)^3}$$

$$4.) f(x) = (\sin x)^{\tan x} \Rightarrow \log(f(x)) = \log((\sin x)^{\tan x}) \Rightarrow \log(f(x)) = \tan x \log(\sin x)$$

$$\frac{f'(x)}{f(x)} = \frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x} + \frac{\log(\sin x)}{\cos^2 x} = 1 + \frac{\log(\sin x)}{\cos^2 x} \Rightarrow f'(x) = \left(1 + \frac{\log(\sin x)}{\cos^2 x}\right) (\sin x)^{\tan x}$$

$$\log(f'(x)) = \log\left(1 + \frac{\log(\sin x)}{\cos^2 x}\right) + \tan x \log \sin x$$

$$\frac{f''(x)}{f'(x)} = 1 + \frac{\log(\sin x)}{\cos^2 x} + \frac{1}{1 + \frac{\log(\sin x)}{\cos^2 x}} \cdot \left(\frac{1}{\sin x \cos x} - \frac{\sin x \log(\sin x)}{\cos^2 x}\right)$$

$$f''(x) = \left[1 + \frac{\log(\sin x)}{\cos^2 x} + \frac{1}{1 + \frac{\log(\sin x)}{\cos^2 x}} \left(\frac{1}{\sin x \cos x} - \frac{\sin x \log(\sin x)}{\cos^2 x}\right)\right] \cdot \left[1 + \frac{\log(\sin x)}{\cos^2 x}\right] (\sin x)^{\tan x}$$

Function 1: $f(x) = \frac{x}{x^2+1}$.

$$f(0) = 0, \quad f(x > 0) > 0, \quad f(x < 0) < 0 \quad (1)$$

$$x \rightarrow \pm\infty \Rightarrow f(x) \rightarrow 0 \quad (2)$$

Maximum: $\frac{df}{dx} = 0$, $\frac{d^2f}{dx^2} > 0$. Minimum: $\frac{df}{dx} = 0$, $\frac{d^2f}{dx^2} < 0$.

$$\frac{df}{dx} = \frac{1-x^2}{(1+x^2)^2} = 0 \Rightarrow x = \pm 1 \quad (3)$$

$$f_{max} = \frac{1}{2}, \quad f_{min} = -\frac{1}{2} \quad (4)$$

Function 2: $f(x) = \frac{\sin x}{x}$.

$$f(0) = \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad (5)$$

$$f(x \rightarrow \pm\infty) \rightarrow 0 \quad (6)$$

$$\frac{df}{dx} = \frac{x \cos x - \sin x}{x^2} \quad (7)$$

$$\frac{df}{dx} = 0 \Rightarrow \tan x = x \quad (8)$$

The last equation has infinite number of solutions but it is impossible to find analytical solutions.

Function 3: $f(x) = x \exp(-x^2)$.

$$f(0) = 0, \quad f(x > 0) > 0, \quad f(x < 0) < 0 \quad (9)$$

$$f(x \rightarrow \pm\infty) \rightarrow 0 \quad (10)$$

$$\frac{df}{dx} = (1-2x^2)e^{-x^2} \quad (11)$$

$$\frac{df}{dx} = 0 \Rightarrow x = \pm \frac{1}{\sqrt{2}} \quad (12)$$

$$f_{max} = \frac{1}{\sqrt{2}}e^{-1/2}, \quad f_{min} = -\frac{1}{\sqrt{2}}e^{-1/2} \quad (13)$$

Function 4: $f(x) = \frac{x^k}{1+x^k}$, $k > \text{integer}$.

$$f(0) = 0 \quad (14)$$

$$f(x \rightarrow \pm\infty) \rightarrow 1 \quad (15)$$

$$f(x) = 1 - \frac{1}{1+x^k} \quad (16)$$

$$\frac{df}{dx} = \frac{kx^{k-1}}{(1+x^k)^2} \quad (17)$$

$$\frac{df}{dx} = 0 \Rightarrow x = 0 \quad (18)$$

If $k = 2n$ then $f(x) > 0$ for $x \neq 0$. Then $x = 0$ is a minimum and $f_{min} = 0$. If $k = 2n + 1$ the $f(x) > 0$ for $x > 0$ and $f(x) < 0$ for $x < 0$. Then at $x = 0$ the function changes sign and there is no minimum or maximum.