

$$\begin{aligned}\sin(a \pm b) &= \sin a \cos b \pm \sin b \cos a \\ 2 \sin a \cos b &= \sin(a + b) + \sin(a - b) \\ 2 \sin\left(\frac{5\pi}{12} + x\right) \cos\left(\frac{5\pi}{12} - x\right) &= \sin \frac{5\pi}{6} - \sin 2x = 0.5 - \sin 2x \\ \sin 2x + 0.5 - \sin 2x &= 0.5 \Rightarrow \text{any } x\end{aligned}$$

③ חשב את הנגזרת והשלים $f(x)$

הפונקציה הבאה:

$$f(x) = \frac{1+x}{1-x} \quad .1$$

$$f(x) = x^2 e^{8x + \cos x} \quad .2$$

$$f(x) = 8(x \ln x - x)^3 \quad .2$$

$$f(x) = x^x \quad .3$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx} a^x = a^x \ln a, \quad \frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} fg = f \frac{d}{dx} g + g \frac{d}{dx} f$$

$$\frac{d}{dx} f(g) = f' \frac{d}{dx} g$$

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$$f(x) = \frac{1+x}{1-x}$$

$$f'(x) = \frac{1}{1-x} - \frac{(1+x)}{(1-x)^2} (-1) = \frac{1}{1-x} + \frac{1+x}{(1-x)^2} = \frac{1-x+1+x}{(1-x)^2} = \frac{2}{(1-x)^2}$$

$$f''(x) = \frac{-4}{(1-x)^3} (-1) = \frac{4}{(1-x)^3}$$

$$f(x) = x^2 e^{\sin x + \cos x}$$

$$f'(x) = 2x e^{\sin x + \cos x} + x^2 e^{\sin x + \cos x} (\cos x - \sin x) = e^{\sin x + \cos x} (2x + x^2 (\cos x - \sin x))$$

$$f''(x) = e^{\sin x + \cos x} (2x + x^2 (\cos x - \sin x)) (\cos x - \sin x) + e^{\sin x + \cos x} (2 + 2x (\cos x - \sin x) - x^2 (\sin x + \cos x))$$

$$f(x) = 8(x \ln x - x)^3$$

$$f'(x) = 24(x \ln x - x)^2 (1 + \ln x - 1) = 24(x \ln x - x)^2 \ln x$$

$$f''(x) = 48(x \ln x - x) (\ln x)^2 + \frac{24}{x} (x \ln x - x)^2$$

$$f(x) = x^x$$

$$f'(x) = x x^{x-1} + x^x \ln x = x^x (1 + \ln x)$$

$$f''(x) = x^x (1 + \ln x)^2 + x^x \left(\frac{1}{x}\right) = x^x \left((1 + \ln x)^2 + \frac{1}{x} \right)$$

Taylor expansions (actually, expansion around 0 are called Maclaurin expansions):

a. $\sin(x) \approx x - \frac{x^3}{6}$

b. $\cos(x) \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$

c. $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

d. $\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

e. $(1+x)^\alpha \approx 1 + \alpha x + \frac{1}{2}\alpha(\alpha-1)x^2 + \frac{1}{6}\alpha(\alpha-1)(\alpha-2)x^3 + \frac{1}{24}\alpha(\alpha-1)(\alpha-2)(\alpha-3)x^4$