e_01_05_012.5

פתרון-

$\vec{v}_2 = \hat{\imath} + 4\hat{\jmath} + 3\hat{k}$ $\qquad$ $\vec{V}_1 = 6\hat{\imath} + 2\hat{k}$

$\vec{V}_1 + \vec{V}_2 + \vec{V}_3 = 0$ $\qquad$?מה ה- $\qquad$ $\vec{V}_3 = ?$ (א

$$\vec{V}_1 + \vec{V}_2 = -\vec{V}_3$$

$$\hat{\imath} + 4\hat{\jmath} + 3\hat{k} + 6\hat{\imath} + 2\hat{k} = -\vec{V}_3$$

$$-7\hat{\imath} - 4\hat{\jmath} - 5\hat{k} = \vec{V}_3$$

ולכן זה כתוב

$$\vec{V}_3 = (-7, -4, -5)$$

$\vec{V}_1 - \vec{V}_2 + \vec{V}_4 = 0$ $\qquad$?מה ה- $\qquad$ $\vec{V}_4 = ?$ (ב

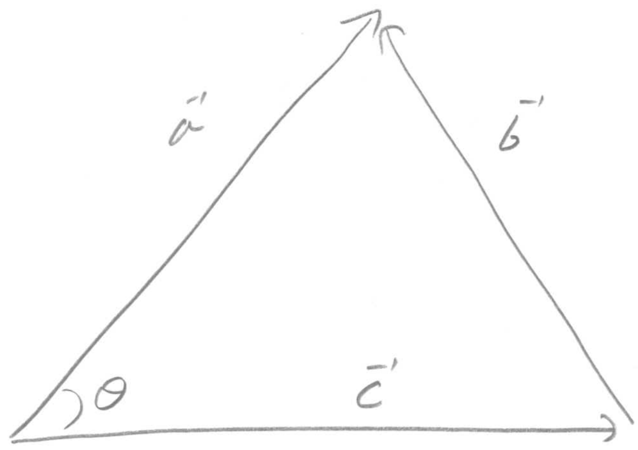
$$\vec{V}_2 - \vec{V}_1 = \vec{V}_4$$

$$\hat{\imath} + 4\hat{\jmath} + 3\hat{k} - 6\hat{\imath} - 2\hat{k} = \vec{V}_4$$

$$-5\hat{\imath} + 4\hat{\jmath} + \hat{k} = \vec{V}_4$$

$\vec{V}_4 = (-5, 4, 1)$ ובכתיבה אחרת-

e_01_2_008



$$\vec{a}\cdot\vec{c} = a\,c\,\cos\theta$$

מכפלה סקלרית!

a הגדל של $|a|$

וכו'

$$\vec{a} = \vec{c} + \vec{b}$$

$$\vec{b} = \vec{a} - \vec{c}$$

עפ״י הגדרה!

נכפיל שני הצדדים ב $\vec{b}$

$$\vec{b}\cdot\vec{b} = (\vec{a}-\vec{c})\cdot(\vec{a}-\vec{c}) = \vec{a}\cdot\vec{a} + \vec{c}\cdot\vec{c} - 2\,\vec{a}\cdot\vec{c}$$

עפ״י הגדרה של מכפלה סקלרית

$$\vec{a}\cdot\vec{a} = |a||a|\cos 0 = |a|^2$$

$$b^2 = a^2 + c^2 - 2\,\vec{a}\cdot\vec{c}$$

וכן

$$b^2 = a^2 + c^2 - 2ac\cos\theta$$

exercise 1_1405

a.

$$\overrightarrow{A} \cdot \vec{B} = 3 \cdot 6 + 4 \cdot (-8) = -14 \tag{1}$$

$$\alpha = \arccos(\frac{\overrightarrow{A} \cdot \vec{B}}{\left\|\overrightarrow{A}\right\| \cdot \left\|\overrightarrow{B}\right\|}) = \arccos(\frac{-14}{\sqrt{3^2+4^2} \cdot \sqrt{6^2+(-8)^2}}) = \arccos(-\frac{7}{25}) \approx 106.26^o \tag{2}$$

b.

$$\overrightarrow{A} \times \vec{B} = det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 6 & -8 & 0 \end{pmatrix} = -48\hat{k} \tag{3}$$

$$\left\|\overrightarrow{A} \times \vec{B}\right\| = 48 \tag{4}$$

c.

$$\overrightarrow{C} \times \vec{D} = det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 3 \\ 2 & 1 & 3 \end{pmatrix} = (3 \cdot 3 - 1 \cdot 3)\hat{i} + (3 \cdot 2 - 3 \cdot 3)\hat{j} + (3 \cdot 1 - 2 \cdot 3)\hat{k} = (6, -3, -3)$$

$$\left\|\overrightarrow{C} \times \vec{D}\right\| = \sqrt{6^2 + (-3)^2 + (-3)^2} \approx 7.34 \tag{5}$$

vectors

given: $\vec{a} = 2\hat{i} - 2\hat{j} - \hat{k}, \vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}, \vec{c} = 4\hat{i} - 1\hat{j} + 3\hat{k}$

A.$|\vec{a}| = \sqrt{2^2 + (-2)^2 + (-1)^2} = 3$;

$|\vec{b}| = \sqrt{6^2 + (-3)^2 + 2^2} = 7$;

$|\vec{c}| = \sqrt{4^2 + (-1)^2 + 3^2} = \sqrt{26}$;

B. $\vec{a} + \vec{b} + \vec{c}$ angles :$\alpha = between\ \vec{a}$; $\beta = between\ \vec{b}$; $\gamma = between\ \vec{c}$;

$\vec{a} + \vec{b} + \vec{c} = (2 + 6 + 4) \cdot \hat{i} + (-2 - 3 - 1) \cdot \hat{j} + (-1 + 2 + 3) \cdot \hat{k} = (12, -6, 4)$;

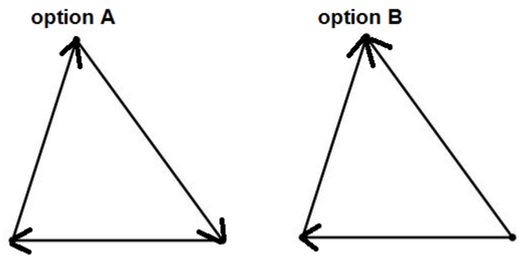
$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{12^2 + (-6)^2 + 4^2} = 14$;

$\cos(\alpha) = \frac{12 \cdot 2 + (-6) \cdot (-2) + 4 \cdot (-1)}{14 \cdot 3} = \frac{16}{21}$; $\boxed{\alpha = 40.37^\circ}$;

$\cos(\beta) = \frac{12 \cdot 6 + (-6) \cdot (-3) + 4 \cdot 2}{14 \cdot 7} = 1$; $\boxed{\beta = 0^\circ}$;

$\cos(\gamma) = \frac{12 \cdot 4 + (-6) \cdot (-1) + 4 \cdot 3}{14 \cdot \sqrt{26}} = 0.92$; $\boxed{\gamma = 22.4^\circ}$;

C. There are 2 options for vector triangle:



option A : head to tail : $\vec{a} + \vec{b} + \vec{c} = (0, 0, 0)$.

and also: $(180^\circ - \theta_{a\,b}) + (180^\circ - \theta_{b\,c}) + (180^\circ - \theta_{a\,c}) = 180^\diamond$.

not our case $(\vec{a} + \vec{b} + \vec{c} = (12, -6, 4))$;

option B: 2 vector addition gives the third vector $\vec{a} + \vec{c} = \vec{b}$.

and also $\theta_{a\,b} + \theta_{b\,c} + (180^\circ - \theta_{a\,c}) = 180^\circ$

$\vec{a} + \vec{c} = (2 + 4) \cdot \hat{i} + (-2 - 1) \cdot \hat{j} + (-1 + 3) \cdot \hat{k} = (6, -3, 2) = \vec{c}$

$\vec{b}$ is in the same direction as $\vec{a} + \vec{b} + \vec{c}$.

therefore: 1. angle between $\vec{a}$ and $\vec{b}$ equal 40.37°; 2. angle between $\vec{c}$ and $\vec{b}$ equal 22.4°.

calculate angle between $\vec{a}$ and $\vec{c}$:

$\cos(\theta) = \frac{2 \cdot 4 + (-2) \cdot (-1) + (-1) \cdot 3}{3 \cdot \sqrt{26}} = 0.458$; $\theta = 62.767^\circ$

$40.37^\circ + 22.4^\circ + (180 - 62.767^\circ) = 180^\circ$

triangle!!!

$$\vec{b}(\vec{a}\cdot\vec{c})-\vec{c}(\vec{a}\cdot\vec{b}) = 3(3,3,1)-2(2,-2,-1) =$$ 1c

$$= (5,13,5)$$

$$\vec{a}\times(\vec{b}\times\vec{c}) = \Big(a_y(\vec{b}\times\vec{c})_z - a_z(\vec{b}\times\vec{c})_y\,,\ a_z(\vec{b}\times\vec{c})_x - a_x(\vec{b}\times\vec{c})_z\,,$$

$$a_x(\vec{b}\times\vec{c})_y - a_y(\vec{b}\times\vec{c})_x\Big) = \Big(a_y(b_xc_y-b_yc_x)-a_z(b_zc_x-b_xc_z),$$
$$a_z(b_yc_z-b_zc_y)-a_x(b_xc_y-b_yc_x),$$
$$a_x(b_zc_x-b_xc_z)-a_y(b_yc_z-b_zc_y)\Big) =$$

$$= (5,13,5)$$

$$\vec{b}(\vec{a}\cdot\vec{c})-\vec{c}(\vec{a}\cdot\vec{b}) = 5(-1,1,3)-5(2,-1,1) =$$ 2

$$(-15,10,10)$$