

? Y_e^m $\int \int \dots$

$$P_1(x) = x$$

$$\frac{dP_1}{dx} = 1$$

$$P_1'(x) = \sqrt{1-x^2} \frac{dP_1}{dx} = \sqrt{1-x^2}$$

$$P_1^0(x) = x$$

$$Y_1^1 = C_1^1 P_1^1(\cos\theta) e^{i\phi} = C_1^1 \sin\theta e^{i\phi}$$

$$Y_1^{-1} = C_1^{-1} \sin\theta e^{-i\phi}$$

$$Y_1^0 = C_1^0 \cos\theta$$

$$\int (Y_1^m)^* Y_1^m d\Omega = 1$$

$\int \sin\theta$

$$\Rightarrow \int_0^{2\pi} d\phi \int_0^{\pi} (C_1^0)^2 \cos^2\theta d\theta = 1$$

$$C_1^0 = \sqrt{\frac{3}{4\pi}}$$

$\int \cos\theta$

$$C_1^1 = C_1^{-1} = \sqrt{\frac{3}{8\pi}}$$

$\int \sin\theta \cos\theta$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$\int \cos^2\theta$

$$Y_1^1 = \sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi}$$

$$Y_1^{-1} = \sqrt{\frac{3}{8\pi}} \sin\theta e^{-i\phi}$$