2. נתון

$$c^2k^2 = \omega^2\left(1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}\right)$$

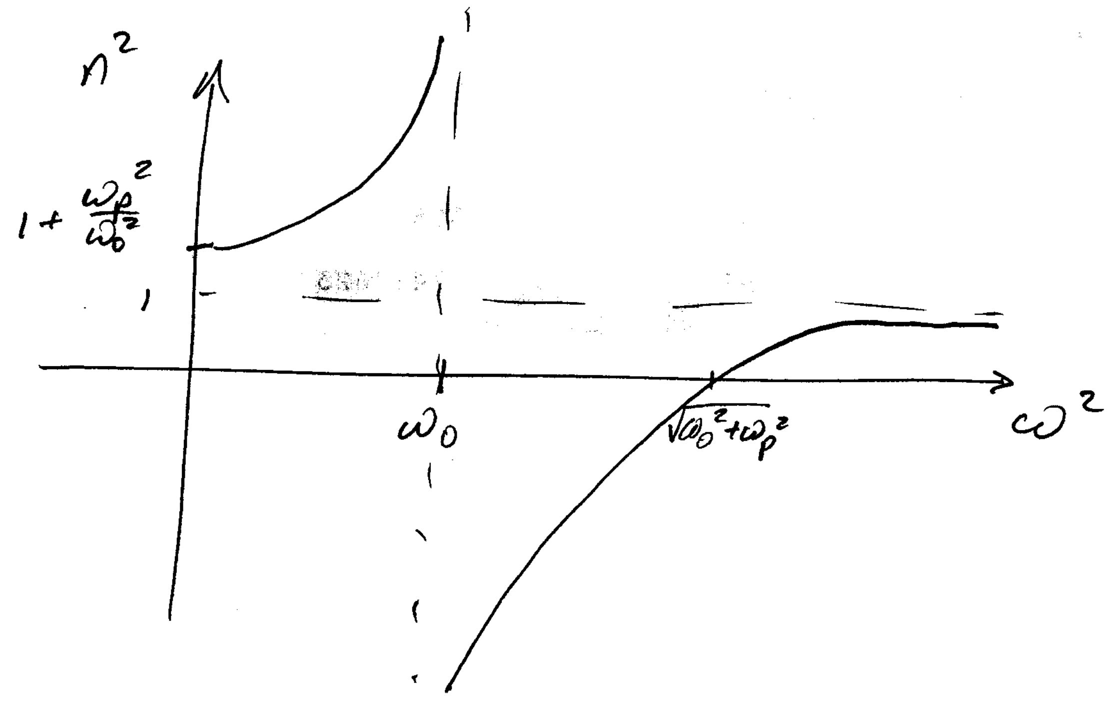
מכיוון ש

$$\frac{\omega}{k} = V_p = \frac{c}{n}$$

אז

$$n = \frac{ck}{\omega}$$

$$n^2 = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}$$



$$\frac{c}{V_g} = n + \omega \frac{dn}{d\omega} = n + \frac{\omega}{2n}\frac{dn^2}{d\omega}$$

$$\Rightarrow n + \frac{\omega}{2n}\frac{dn^2}{d\omega} = 1 + \frac{\omega_p^2}{\omega_0^2-\omega^2} + \frac{\omega}{2n}\frac{\omega_p^2}{(\omega_0^2-\omega^2)^2}(-1)(-2\omega)$$

$$= n + \frac{\omega^2}{(\omega_0^2-\omega^2)}\frac{(n^2-1)}{n}$$

$$\frac{c}{V_g} = \frac{n^2(\omega_0^2-\omega^2) + \omega^2(n^2-1)}{n(\omega_0^2-\omega^2)}$$

$$= \frac{n^2\omega_0^2 - \omega^2}{n(\omega_0^2-\omega^2)}$$

$$= \frac{(n^2-1)\omega_0^2 + (\omega_0^2-\omega^2)}{n(\omega_0^2-\omega^2)}$$

$$\left(\frac{V_g}{c}\right)^2 = \frac{n^2}{\left(1 + \frac{(n^2-1)\,\omega_0^2}{(\omega_0^2-\omega^2)^2}\right)^2}$$

$$= \frac{1 + \frac{\omega_p^2}{\omega_0^2-\omega^2}}{\left(1 + \frac{\omega_p^2\,\omega_0^2}{(\omega_0^2-\omega^2)^2}\right)^2}$$