



$$U = \frac{1}{2} k \left( (\psi_2 - \psi_1)^2 + (\psi_3 - \psi_2)^2 + (\psi_3 - \psi_1)^2 \right)$$

$$\begin{cases} m \ddot{\psi}_1 = +k(\psi_2 - \psi_1) + k(\psi_3 - \psi_1) \\ m \ddot{\psi}_2 = -k(\psi_2 - \psi_1) + k(\psi_3 - \psi_2) \\ m \ddot{\psi}_3 = -k(\psi_3 - \psi_2) - k(\psi_3 - \psi_1) \end{cases}$$

$$\alpha \equiv \frac{k}{3m}$$

$$\begin{pmatrix} \ddot{\psi}_1 \\ \ddot{\psi}_2 \\ \ddot{\psi}_3 \end{pmatrix} = -\alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 \\ -1 & 2-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda) \left( (2-\lambda)^2 - 1 \right) + (-2 + \lambda - 1) - (1 + 2 - \lambda) = 0$$

$$(2-\lambda) \left( (2-\lambda-1)(2-\lambda+1) \right) + \lambda - 3 - 3 + \lambda = 0$$

$$(2-\lambda)(3-\lambda)(1-\lambda) + 2(\lambda-3) = 0$$

$$(3-\lambda)(\lambda - 2\lambda - \lambda + \lambda^2 - \lambda) = 0$$

$$\lambda_{1,2} = 3 \quad \lambda_3 = 0$$

... για τον κύριο φυσικό  $\psi_1 = \psi_2 = \psi_3$   $\vec{r}'' = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \leftarrow \omega^2 = 0 \leftarrow \lambda = 0$

... για τον κύριο φυσικό  $\vec{r}'' = \omega^2 = \frac{3k}{m} \leftarrow \lambda = 3$

$$\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \text{ οι } \vec{r}''$$

$$\psi_3 = -2\psi_1 = -2\psi_2 \quad \psi_1 = -\psi_2 \quad \psi_3 = 0$$