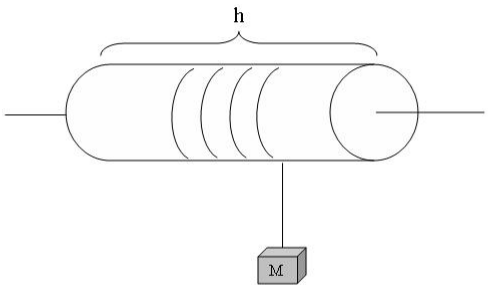
Induction

Submitted by: I.D. 303689129

The problem:

A surface of an isolating cylinder, with a radius R and A length h, is charged with a charge Q. The charge is distributed uniformly on the surface of the cylinder. A mass m is wrapped around the center of the cylinder. Due to the force of gravity the mass is falling down, which rotates the cylinder around its axis. Find the acceleration of the mass ignoring the cylinder's moment of inertia.



The solution:

Let the cylinder stand on the XY ($\rho\varphi$) plane and rotate around the Z axis in the direction of φ. We can assume that an induced electromagnetic force may cause a torque along the rotation axis. Therefore, the torque equation would be:

$$\sum \vec{\tau} = \vec{R} \times m\vec{a} = \vec{R} \times m\vec{g} + \vec{R} \times \vec{F_q} \tag{1}$$

Now we shall evaluate the induced electromagnetic force.
The charge density on the cylinder surface is given by:

$$\sigma = \frac{Q}{2\pi Rh} \tag{2}$$

The charge is rotating, therefore the current density will be:

$$\vec{J} = \sigma\vec{v} = \sigma(\omega \times R) \tag{3}$$

and the total current on the cylinder is:

$$I = \int_h \vec{J} \cdot ds = \sigma \left\|(\omega \times R)\right\| h \tag{4}$$

Using the Ampere's law and taking the same considerations as with a coil, the magnetic field inside of the cylinder is:

$$\oint \vec{B} \cdot d\vec{l} = 4\pi K I_{in} \Rightarrow B\hat{z} = \frac{4\pi K I}{h}\hat{z} = 4\pi K\sigma\omega R\hat{z} \tag{5}$$

And the total flux through the cylinder is:

$$\Phi = 4\pi K\sigma\omega R\hat{z} \cdot \pi R^2\hat{z} \tag{6}$$

Now, using the Faraday's Law we can calculate the electric field along the cylinder's surface:

$$\oint_{2\pi R} E_{\hat{\varphi}} dl = -\dot{\Phi} \Rightarrow 2\pi R E\hat{\varphi} = -\frac{\partial\left[4\pi^2 K\sigma\omega R^3\right]}{\partial t} = -4\pi^2 K\sigma\dot{\omega}R^3 \tag{7}$$

$$E\hat{\varphi} = -2\pi K\sigma R^2\dot{\omega}\hat{\varphi} \tag{8}$$

The total elctromagnetic force can be calulated by integrating the electric field on each charge on the cylinder's surface:

$$\vec{F}_q = \int\limits_S E\hat{\varphi}dq = -2Q\pi K\sigma R^2\dot{\omega}\hat{\varphi} = \frac{-Q^2KR\dot{\omega}}{h}\hat{\varphi} \tag{9}$$

$$\dot{\omega} = \frac{a}{R} \tag{10}$$

$$\vec{F}_q = \frac{-Q^2Ka}{h}\hat{\varphi} \tag{11}$$

Now we can insert equation (11) into equation (1) and evaluate the total torque on the cylinder.

$$\sum \vec{\tau} = Rma\hat{z} = Rmg\hat{z} + R\frac{-Q^2Ka}{h}\hat{z} \tag{12}$$

Therefore the acceleration is:

$$\vec{a} = \frac{m\vec{g}h}{mh + Q^2K} \tag{13}$$