Self Inductance

Submitted by: I.D. 043423755

The problem:
A long coaxial cable is made out of 2 thin metallic tubes with the radii a, b $(b > a)$.
What is the self inductance per length unit?

The solution:

A. One way of solving the problem:

$$L = \frac{\Phi}{I} \tag{1}$$

For $a < r < b$

$$\vec{B} = \frac{(\mu_0)I}{2\pi r}\hat{\theta} \tag{2}$$

and zero otherwise. The magnetic field is tangential to the direction θ, so that the area element is ldr.

$$d\Phi(r) = B(r)ldr \tag{3}$$

$$\Phi = \int_a^b \frac{\mu_0 Il}{2\pi r}dr = \frac{\mu_0}{2\pi}Il\ln\frac{b}{a} \tag{4}$$

The ratio L/l is:

$$\frac{L}{l} = \frac{\Phi}{Il} = \frac{\mu_0}{2\pi}\ln\frac{b}{a} \tag{5}$$

B. Second way of solving the problem
Let's define the density of magnetic energy

$$u = \frac{1}{2\mu_0}B^2 = \frac{1}{2\mu_0}\frac{\mu_0^2 I^2}{4\pi^2 r^2} \tag{6}$$

We find the total energy stored in the system

$$U = \frac{1}{2}LI^2 \tag{7}$$

And find L from there

$$U = \int udV = \int_0^l \int_0^{2\pi} \int_a^b \frac{\mu_0^2 I^2}{2\mu_0 4\pi^2 r^2} r dr d\theta dz = \frac{1}{2}\frac{\mu_0 l}{2\pi}I^2\ln\frac{b}{a} \tag{8}$$

Hence

$$\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{b}{a} \tag{9}$$