Mutual Inductance

Submitted by: I.D. 043423755

The problem:
Two loops of radius a are at a distance b from each other, such that the planes of the loops are parallel, and perpendicular to the axis connecting them.

1. Assuming $b \gg a$, what is the mutual inductance of the system?
2. In one loop there is a constant current I and the other loop rotates at the angular velocity along its diameter. What is the induced EMF in the rotating loop?

The solution:

The method: We assume that there is a changing current in one of the loops and we will calculate the e.m.f in the second loop and thus extract the inductance coefficient.
1. The magnetic field $\vec{B}$ that is created along the z axis of the ring

$$B = \frac{(\mu_0 I)a^2}{2(a^2+b^2)^{3/2}}\hat{z} \tag{1}$$

$b \gg a$ - meaning that inside the area of the second ring, the magnetic field is approximately the same. Using the Taylor expansion.

$$B = \frac{\mu_0 I a^2}{2b^3(1+(a/b)^2)^{3/2}} = \frac{\mu_0 I a^2}{2b^3} + O((a/b)^2) \tag{2}$$

Let's calculate the flux

$$\Phi = B \cdot A = \frac{\mu_0 I a^2}{2b^3}\pi a^2 = \frac{\mu_0 \pi I a^4}{2b^3} \tag{3}$$

Now we use the Faraday's law

$$\epsilon = -\dot{\Phi} = -\frac{\mu_0 \pi a^4}{2b^3}\dot{I} \tag{4}$$

Finally the mutual inductance coefficient M is the expression multiplying the time derivative of the current.

$$M = \frac{\mu_0 \pi a^4}{2b^3} \tag{5}$$

2. The magnetic flux through the loop now changes because the area changes with time

$$A = \pi a^2 \cos(\omega t) \tag{6}$$

Thus

$$\Phi = B \cdot A = \frac{\mu_0 \pi a^4 I(t)}{2b^3}\cos(\omega t) \tag{7}$$

$$\epsilon = -\dot{\Phi} = -\frac{\mu_0 \pi a^4}{2b^3}(\dot{I}\cos(\omega t) - \omega I \sin(\omega t)) \tag{8}$$