Magnetic Field and Force

Submitted by: I.D. 310634175

The problem:

A rod of the length L and current I is turned over to a ring.

1. Find the maximal torque $\vec{N}$ which acts on the ring and find the angle α.
2. Find the maximal flux which flows through the ring. Find the angle α .
3. When $\alpha = 30°$, find the torque on the ring and the flux which flows through the ring.
4. How the flux and torque would change if the ring would consist of two windings?

The solution:

1. The magnetic moment dipole is $\vec{m} = I\vec{A}$ and the torque is $\vec{N} = \vec{m} \times \vec{B}$

$$2\pi R = L \quad (1)$$

$$A = 2\pi R \Rightarrow A = \frac{L^2}{4\pi} \quad (2)$$

$$\vec{N} = \vec{m} \times \vec{B} \quad (3)$$

$$N(\alpha) = I\frac{L^2}{4\pi}B\sin\alpha \quad (4)$$

$$N_{\max} = \frac{IL^2B}{4\pi} \text{ for } \alpha = \frac{\pi}{2} \quad (5)$$

The direction of the torque is perpendicular to the magnetic field and to the magnetic moment.
2.The flux is:

$$\Phi = \int \vec{B} \cdot d\vec{a} = B\pi R^2 \cos\alpha \quad (6)$$

$$\Phi(\alpha) = \frac{BL^2}{4\pi}\cos\alpha \quad (7)$$

$$\Phi_{\max}(\alpha = 0) = \frac{BL^2}{4\pi} \quad (8)$$

3.For $\alpha = \pi/6$

$$N_{(\alpha=\frac{\pi}{6})} = I \cdot \frac{L^2}{4\pi} \cdot B \cdot \sin\frac{\pi}{6} = I\frac{L^2}{8\pi} \cdot B \quad (9)$$

$$\Phi_{(\alpha=\frac{\pi}{6})} = \sqrt{3}\frac{BL^2}{8\pi} \quad (10)$$

4.A new radius is :

$$2(2\pi R^{'}) = L \Rightarrow R^{'} = \frac{L}{4\pi} \quad (11)$$

A new area is:

$$A^{'} = \pi R^{'2} = \frac{L^2}{16\pi} \quad (12)$$

Therefore:

$$
\begin{aligned}
N &= 2I\frac{L^2}{16\pi}\cdot B\sin\alpha && (13)\\
N_{\max} &= I\frac{L^2}{8\pi}B && (14)\\
\Phi &= 2\int \vec{B}\cdot d\vec{a} = B\pi R_1^2\cdot\cos\alpha = \frac{BL^2}{8\pi}\cos\alpha && (15)\\
\Phi_{\max} &= \frac{L^2B}{8\pi} && (16)
\end{aligned}
$$

The factor of 2 is due to the fact that there are two windings.