Magnetic Field

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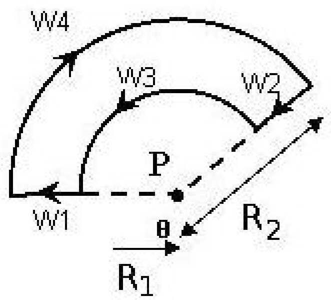
The problem:

Calculate the magnetic field in the point P which is the center of part of a ring with radii R_1,R_2 given $I = 3.6\ A$, $R_1 = 17\ cm$, $R_2 = 45\ cm$, $\theta = 2\pi/3$.



The solution:

We will use cylindrical coordinates with the origin at P, $\hat{\varphi}$ going counter-clockwise such that $\varphi = 0$ at R_2 meaning $\hat{z}$ is facing from the sheet.
The requested field is the superposition of four wire-parts W_1, W_2, W_3, W_4. Using the Biot-Savarts law we can calculate the contribution of each wire-parts.
For W_1, W_2:
The wire position is $\vec{r}' = r\hat{r}$ and we calculate the field at the origin so $\vec{r} = 0$, therefore, $\vec{r} - \vec{r}' = -r\hat{r}$. W_1, W_2 dont contribute to the field because:

$$\vec{dl} = d\vec{r} \tag{1}$$

$$\vec{dl} \times (\vec{r} - \vec{r}') = dr\hat{r} \times (-r\hat{r}) = 0 \tag{2}$$

$$\vec{B}_1 = \vec{B}_2 = 0 \tag{3}$$

For W_3, W_4:

$$\vec{dl_i} = R_i d\varphi\hat{\varphi} \tag{4}$$

$$(\vec{r} - \vec{r}_i') = -R_i\hat{r} \tag{5}$$

$$\vec{dl_i} \times (\vec{r} - \vec{r}_i') = R_i^2 d\varphi\hat{z} \tag{6}$$

and therefore:

$$\vec{B}_1 = KI\int_0^\theta \frac{\vec{dl_1} \times (\vec{r} - \vec{r}_1')}{(\vec{r} - \vec{r}_1')} = KI\int_0^\theta \frac{R_1^2 d\varphi}{R_i^3}\hat{z} = \frac{KI\theta}{R_1}\hat{z} \tag{7}$$

$$\vec{B}_2 = KI\int_\theta^0 \frac{\vec{dl_1} \times (\vec{r} - \vec{r}_1')}{(\vec{r} - \vec{r}_1')} = KI\int_\theta^0 \frac{R_2^2 d\varphi}{R_2^3}\hat{z} = -\frac{KI\theta}{R_2}\hat{z} \tag{8}$$

and the solution is:

$$\vec{B}_P = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = KI\theta\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\hat{z} \tag{9}$$

substituting numbers:

$$B = KI\theta\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{\mu_0}{4\pi} \cdot 3.6 \cdot \frac{2}{3}\pi\left(\frac{1}{0.17} - \frac{1}{0.45}\right) = 2.76\ \mu T \tag{10}$$