Electric Current

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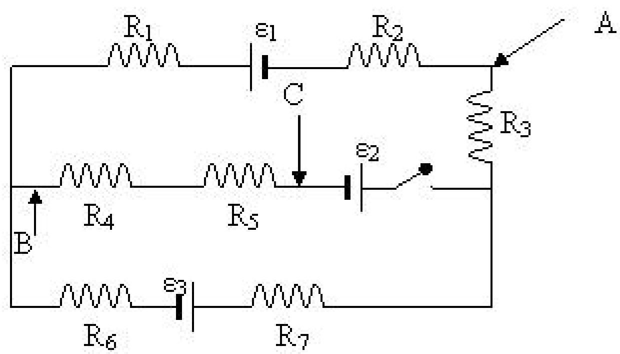
The problem:

Given the values: $\varepsilon_1 = 1\,V, \varepsilon_2 = 0.5\,V, \varepsilon_3 = 0.6\,V, R_1 = R_2 = 0.5\,\Omega, R_3 = 1\,\Omega, R_4 = 0.4\,\Omega,$
$R_5 = R_6 = 0.6\,\Omega, R_7 = 0.7\,\Omega$

1. Calculate the current flowing through each resistor, and the potential difference between B and A when the switch is open.
2. Calculate the current flowing through each resistor, and the potential difference between B and C when the switch is closed.



The solution:

1. Since the switch is open we can disregard the middle branch of the circuit, leaving only a circuit with resistors and voltage sources in series. By using Ohm's Law we find:

$$V_t = \varepsilon_1 + \varepsilon_3 = 1 + 0.6 = 1.6\,V \quad (1)$$
$$R_t = R_1 + R_2 + R_3 + R_6 + R_7 = 0.5 + 0.5 + 1 + 0.6 + 0.7 = 3.3\,\Omega \quad (2)$$
$$I = \frac{V_t}{R_t} = \frac{\varepsilon_1 + \varepsilon_3}{R_1 + R_2 + R_3 + R_6 + R_7} = \frac{1.6}{3.3} = 0.485\,A \quad (3)$$

With the current moving counter clockwise because of the direction of the voltage sources.

Now in order to calculate the potential difference between B and A, all we have to do is calculate the voltage between these points:

$$V_{BA} = \varepsilon_1 - I(R_1 + R_2) = 1 - I = 0.515\,V \quad (4)$$

2. Now that the switch is closed we can no longer disregard the middle brance, and we have to use Kirchhoff's laws.
We will choose the right node as our junction and assume that I_1 comes from above, I_2 from below and I_3 flows to the left.
Now, our first path will be clockwise through $R_1, \varepsilon_1, R_2, R_3, \varepsilon_2, R_5, R_4$, and our second will be likewise clockwise through $R_1, \varepsilon_1, R_2, R_3, R_7, \varepsilon_3, R_6$. giving us the following equations:

$$I_1 + I_2 = I_3 \quad (5)$$
$$I_1R_1 + \varepsilon_1 + I_1R_2 + I_1R_3 + \varepsilon_2 + I_3R_5 + I_3R_4 = 0 \quad (6)$$
$$I_1R_1 + \varepsilon_1 + I_1R_2 + I_1R_3 - I_2R_7 + \varepsilon_3 - I_2R_6 = 0 \quad (7)$$

Using simple math, we found the currents: $I_1 = -0.6\ A, I_2 = 0.3\ A$ and $I_3 = -0.3\ A$, while the negative sign signals that the direction in which these currents flow is opposite to the one we chose. in order to find the potential difference between points B and C we calculate the voltage, keeping in mind that I_3 now has a new direction. thus:

$$V_{BC} = I_3(R_4 + R_5) = 0.3(0.4 + 0.6) = 0.3\,V \tag{8}$$