

Conductors

Because of the symmetry $P_{11} = P_{22} = P_{33} \equiv A$, $P_{12} = P_{13} = P_{23} \equiv B$.

One has at every step

$$\begin{aligned}\phi'_1 &= P_{11}q'_1 + P_{12}q'_2 + P_{13}q'_3 \\ &= Aq'_1 + B(q'_2 + q'_3)\end{aligned}$$

$$\phi'_2 = Aq'_2 + B(q'_1 + q'_3)$$

$$\phi'_3 = Aq'_3 + B(q'_1 + q'_2)$$

First step: $\phi_1 = \phi_0$, $q'_1 = q_1$, $\phi_2 = Bq_1$, $\phi_3 = Bq_1$, where ϕ_0 is the potential of the distant conductor.

Since $q'_2 = q'_3 = 0$ one has $\phi_1 = \phi_0 = Aq_1$. Second step: $\phi_2 = \phi_0$, $q'_1 = q_1$ (the charge remains on the conductor after disconnecting it from the distant one), $q'_2 = q_2$.

At this step

$$\phi_2 = \phi_0 = Aq_2 + Bq_1$$

Third step similarly:

$$\phi_3 = \phi_0 = Aq_3 + B(q_1 + q_2)$$

One has

$$Aq_1 = Aq_2 + Bq_1$$

$$Aq_1 = Aq_3 + B(q_1 + q_2)$$

$$B = A(1 - q_2/q_1)$$

$$q_3 = q_1 + \frac{q_2^2 - q_1^2}{q_1} = \frac{q_2^2}{q_1}$$