# Capacitors

**The problem:**

A cylindrical capacitor of inner radius $R_1$ and outer radius $R_2$ and length $L \gg R_2$ (may be treated as infinite) contains dielectric material such that the dielectric constant is given by:

$$\epsilon = \begin{cases} \epsilon_1, & 0 < \theta < \pi \\ \epsilon_2, & \pi < \theta < 2\pi \end{cases}$$

1. Find the capacitance of the system by calculating the ratio of the charge to the potential difference.
2. The charge on each cylinder is $\pm q$. What is the energy of the system?

**The solution:**

Let the inner cylinder have the charge $q$ and the outer one have the charge $-q$. Building a Gaussian cylindrical shell of a radius $r$ and a height $h$ we get:

$$\int \varepsilon \vec{E} \cdot d\vec{A} = 4\pi k q \tag{1}$$

$$\varepsilon_1 E\ \pi r h + \varepsilon_2 E\ \pi r h = 4\pi k q \frac{h}{L} \tag{2}$$

$$E = \frac{4kq}{(\varepsilon_1 + \varepsilon_2) r L} \tag{3}$$

and we know that: $V = \int \vec{E} \cdot d\vec{l}$:

$$V = \int \vec{E} \cdot d\vec{l} = \int_{R_1}^{R_2} \frac{4kq}{(\varepsilon_1 + \varepsilon_2) r L} dr = \frac{4kq}{(\varepsilon_1 + \varepsilon_2) L} \ln \frac{R_2}{R_1} \tag{4}$$

Finally we get:

$$C = \frac{q}{V} = \frac{(\varepsilon_1 + \varepsilon_2) L}{4k \ln \frac{R_2}{R_1}} \tag{5}$$

The energy of the system is

$$U = \frac{1}{2} q V = \frac{2kq^2}{(\varepsilon_1 + \varepsilon_2) L} \ln \frac{R_2}{R_1} \tag{6}$$