The electric field

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The problem:

An electrical infinite straight wire with charge density λ is at the center of an infinite hollow cylinder with charge density σ and a radius a.
What is the electrical field everywhere?

The solution:

The solution is superposition of two fields: one from wire and another one from cylinder. Both fields we will find with the Gauss' law. Because of symmetry,the field are in the radial direction only.

Field of the wire:

$$\begin{aligned} E \cdot 2\pi r l &= 4\pi k Q = 4\pi \lambda l && (1) \\ E &= \frac{2\lambda k}{r} && (2) \end{aligned}$$

Field of the cylinder:
inside:

$$\begin{aligned} E \cdot 2\pi r l &= 4\pi k Q = 4\pi 0 && (3) \\ E &= 0 && (4) \end{aligned}$$

outside:

$$\begin{aligned} E \cdot 2\pi r l &= 4\pi k Q = 4\pi k (2\pi a l)\sigma && (5) \\ E &= \frac{4\pi k a \sigma}{r} && (6) \end{aligned}$$

The field from the whole system:

$$\vec{E} = \begin{cases} \frac{2\lambda k}{r}\hat{r}, & r < R \\ \frac{2\lambda k}{r}\hat{r} + \frac{4\pi a\sigma}{r}\hat{r}, & r > R \end{cases} \quad (7)$$