Electric field

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The problem:

- Calculate directly the electric field flux through an infinite plane a distance d from a point charge.
- Explain why your result does not depend on d.

The solution:

Let the plane to be at the $x-y$ plane, so that any point on it is $\vec{r} = (r\cos(\varphi), r\sin(\varphi), 0)$ and the point charge is at $\vec{r'} = (0, 0, d)$. By the Coloumb law

$$d\vec{E} = \frac{kdq(\vec{r} - \vec{r'})}{|\vec{r} - \vec{r'}|^3} \tag{1}$$

In our case

$$\vec{r} - \vec{r'} = (r\cos(\varphi), r\sin(\varphi), -d) \tag{2}$$

$$|\vec{r} - \vec{r'}| = (r^2 + d^2)^{1/2} \tag{3}$$

So that

$$\vec{E} = \frac{kq}{(r^2 + d^2)^{3/2}}(r\cos(\varphi), r\sin(\varphi), -d) \tag{4}$$

The flus is

$$\oint \vec{E} \cdot d\vec{A} = \int E_z dA \tag{5}$$

$$= \int_0^{2\pi} d\varphi \int_0^{\infty} r dr \frac{kqd}{(r^2 + d^2)^{\frac{3}{2}}} \tag{6}$$

$$= 2\pi kqd \int_0^{\infty} \frac{r dr}{(r^2 + d^2)^{\frac{3}{2}}} = 2\pi kq \tag{7}$$

The total flux from a point charge if $4\pi kq$. Since the plain is infinite, half of the electric field lines pass through this plane (not depending on d), that is half of the flux $2\pi kq$.