Gauss' law - cylindrical symmetry

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The problem:

In an infinite cylinder of a radius R charged uniformly with a charge density ρ there is a cylindrical infinite hole of a radius r_0 which center positioned at $\vec{b}$. Find the electric field.

The solution:

We use the principle of superposition: the problem is equal to a full cylinder of a radius R charged with ρ plus a full cylinder of a radius r_0 charged with $-\rho$.

1. outside the cylinder

$$E = \frac{2\pi k\rho R^2}{|\vec{r}|^2}\vec{r} + \frac{-2\pi k\rho r_0^2}{|\vec{r}-\vec{b}|^2}(\vec{r}-\vec{b}) \quad (1)$$

2. inside the cylinder, outside the hole

$$E = 2\pi k\rho\vec{r} + \frac{-2\pi k\rho r_0^2}{|\vec{r}-\vec{b}|^2}(\vec{r}-\vec{b}) \quad (2)$$

3. inside the hole

$$E = 2\pi k\rho\vec{r} - 2\pi k\rho(\vec{r}-\vec{b}) = 2\pi k\rho\vec{b} = \text{const!} \quad (3)$$