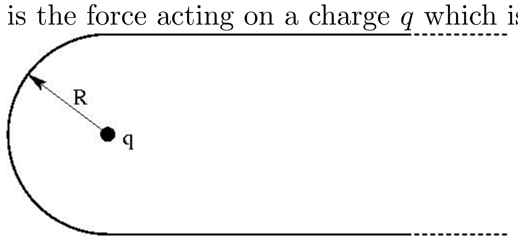
Electric force

The problem:

An infinite wire charged uniformly with the charge density λ is bent as shown on the picture. What is the force acting on a charge q which is put into the center of the half-circle?



The solution:

We divide the problems to 2 parts: (1) finding the force acting from the straight line and (2) finding the force acting from the half circle.
The infinidecimal charge is $dq = \lambda dl$.
For the straight line:

$$\begin{aligned} \vec{r} &= 0\ , \quad \vec{r'} = x\hat{i} + R\hat{j} && (1) \\ dF_x &= -\frac{kQdq}{(x^2+R^2)^{\frac{3}{2}}}x = -\frac{kQ\lambda xdx}{(x^2+R^2)^{\frac{3}{2}}} && (2) \\ dF_y &= -\frac{k\lambda Rdx}{(x^2+R^2)^{\frac{3}{2}}} && (3) \\ F_x &= -2\int_0^\infty \frac{kQ\lambda xdx}{(x^2+R^2)^{\frac{3}{2}}} = -\frac{2\lambda Qk}{R} && (4) \end{aligned}$$

where the last integral could be solved (for example) by the substitution $x = \cos\alpha$. The y-components of the force are canceled since there are 2 wires, and the factor of 2 in the last equation also comes from this fact.
For the half circle:

$$\begin{aligned} \vec{r'} &= R\cos\theta\hat{i} + R\sin\theta\hat{j} && (5) \\ dF_x &= \frac{kQdq}{R^3}R\cos\theta = \frac{kQ\lambda R^2 d\theta}{R^3}\cos\theta && (6) \\ F_x &= \frac{2\lambda Qk}{R} && (7) \end{aligned}$$

Therefore,

$$\sum \vec{F} = 0 \qquad (8)$$