Coulomb law

Submitted by: I.D. 303689129

The problem:

Two identical rods (length L) are placed on the $\hat{x}$ axis. The distance between the rods is L. The rods are charged uniformly, and the total charge on each rod is Q. Find the force on the right rod.

The solution:

The charge density on each rod is

$$\lambda = \frac{Q}{L} \tag{1}$$
$$dq = \lambda dl \tag{2}$$

We can sum the Columb forces between the charges on the left rod and the right rod.

$$\sum \vec{F} = k \sum_i \Delta q_i \sum_j \Delta q_j \frac{(\vec{r}_j - \vec{r}_i)}{(\vec{r}_j - \vec{r}_i)^3} \tag{3}$$

The index i iterates the charges (positioned at $\vec{r}_i$) on the left rod. The index j iterates the charges (positioned at $\vec{r}_j$)on the right rod. The quantity $\vec{r}_j - \vec{r}_i$ gives the distance and the direction between 2 summed charges.
Integrating:

$$F_x = k \int_0^L \lambda dl_1 \int_{2L}^{3L} \lambda dl_2 \frac{1}{(l_2 - l_1)^2} \tag{4}$$
$$= k\lambda^2 \int_0^L dl_1 \int_{2L}^{3L} \frac{dl_2}{(l_2 - l_1)^2} \tag{5}$$
$$= k\lambda^2 \int_0^L dl_1 \left[\frac{1}{l_1 - l_2}\right]_{2L}^{3L} \tag{6}$$
$$= k\lambda^2 \int_0^L \left[\frac{1}{l_1 - 3L} - \frac{1}{l_1 - 2L}\right] dl_1 \tag{7}$$
$$= k\lambda^2 \left[\ln |l_1 - 3L|_0^L - \ln |l_1 - 2L|_0^L\right] \tag{8}$$
$$= k\lambda^2 \ln \frac{4}{3} \tag{9}$$
$$= k\frac{Q^2}{L^2} \ln \frac{4}{3} \tag{10}$$