Scale factors

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The problem:

Find the scale coefficients for:

1. cylindrical coordinates
2. spherical coordinates
3. parabolic coordinates

The solution:

1. $\vec{r}$ in cylindrical coordinates is given by $\vec{r} = \rho\cos\varphi\hat{x} + \rho\sin\varphi\hat{y} + z\hat{z}$.

$$h_\rho = \left|\frac{\partial\vec{r}}{\partial\rho}\right| = |\cos\varphi\hat{x} + \sin\varphi\hat{y}| = \sqrt{\cos^2\varphi + \sin^2\varphi} = 1 \tag{1}$$

$$h_\varphi = \left|\frac{\partial\vec{r}}{\partial\varphi}\right| = |-\rho\sin\varphi\hat{x} + \rho\cos\varphi\hat{y}| = \rho\sqrt{\cos^2\varphi + \sin^2\varphi} = \rho \tag{2}$$

$$h_z = \left|\frac{\partial\vec{r}}{\partial z}\right| = 1 \tag{3}$$

2. $\vec{r}$ in spherical coordinates is given by $\vec{r} = r\sin\theta\cos\varphi\hat{x} + r\sin\theta\sin\varphi\hat{y} + r\cos\theta\hat{z}$.

$$h_r = \left|\frac{\partial\vec{r}}{\partial r}\right| = |\sin\theta\cos\varphi\hat{x} + \sin\theta\sin\varphi\hat{y} + \cos\theta\hat{z}| = 1 \tag{4}$$

$$h_\theta = \left|\frac{\partial\vec{r}}{\partial\theta}\right| = |r\cos\theta\cos\varphi\hat{x} + r\cos\theta\sin\varphi\hat{y} - r\sin\theta\hat{z}| = r \tag{5}$$

$$h_\varphi = \left|\frac{\partial\vec{r}}{\partial\varphi}\right| = |-r\sin\theta\sin\varphi\hat{x} + r\sin\theta\cos\varphi\hat{y}| = r\sin\theta \tag{6}$$

3. $\vec{r}$ in parabolic coordinates is given by $\vec{r} = \frac{1}{2}(u^2 - v^2)\hat{x} + uv\hat{y} + z\hat{z}$.

$$h_u = \left|\frac{\partial\vec{r}}{\partial u}\right| = |u\hat{x} + v\hat{y}| = \sqrt{u^2 + v^2} \tag{7}$$

$$h_v = \left|\frac{\partial\vec{r}}{\partial v}\right| = |-v\hat{x} + u\hat{y}| = \sqrt{u^2 + v^2} \tag{8}$$

$$h_z = \left|\frac{\partial\vec{r}}{\partial z}\right| = 1 \tag{9}$$