RLC circuit

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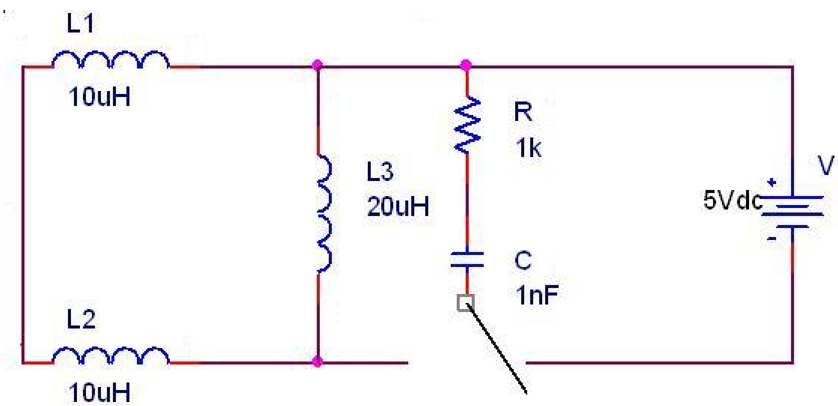
The problem:

For the given RLC circle:

1. find the law of connecting inductances in series and in parallel in general.
   What is the total induction of the given circle?
2. The switch is on the right hand side for $3\tau_{\text{sec}}$ and after is moved to the left side.
   - Find the resonance frequency of the system.
   - find the current through the resistor as a function of time.



The solution:

a. inductance in series:

$$I_1 = I_2 \Rightarrow \varepsilon = (L_1 + L_2)\dot{I}_1 = L_{eff}\dot{I} \Rightarrow L_1 + L_2 = L_{eff} \tag{1}$$

inductance in parallel:

$$I_1 + I_2 = \frac{\varepsilon}{L_1} + \frac{\varepsilon}{L_2} = \frac{\varepsilon(L_1 + L_2)}{L_1 L_2} \Rightarrow \frac{\varepsilon}{I_1 + I_2} = \frac{L_1 L_2}{L_1 + L_2} = L_{eff} \Rightarrow \frac{1}{L_{eff}} = \frac{1}{L_1} + \frac{1}{L_2} \tag{2}$$

in our case:

$$L_{eff} = \frac{(L_1 + L_2) \cdot L_3}{L_1 + L_2 + L_3} \tag{3}$$

after $3\tau(RC)$ we get:

$$V = \varepsilon(1 - e^{\frac{-t}{Rc}}) = \varepsilon(1 - e^{-3}) \approx 4.75v \tag{4}$$

and the equation of the circuit is:

$$\frac{q}{c} + IR + L\dot{I} = 0 \Rightarrow \frac{q}{c} + R\dot{q} + L\ddot{q} = 0 \tag{5}$$

we shall start analysis with the simplest case- R=0, in this case we have:

$$\ddot{q} = -\frac{1}{Lc}q \tag{6}$$

and the solution is:

$$q = Ae^{i\omega_0 t} \tag{7}$$

where

$$\omega_0 = \sqrt{\frac{1}{L_{eff}c}} = 8\frac{2}{3} \cdot 10^6 H_{rz} \tag{8}$$

substitute $q = Ae^{i\omega t}$ in the equation we get:

$$-\omega^2 + \frac{iR}{L}\omega + \omega_0^2 = 0 \Rightarrow \omega = \frac{i}{2}\Gamma \pm \sqrt{\omega_0^2 - \frac{\Gamma^2}{4}} \tag{9}$$

where

$$\Gamma = \frac{R}{L} \tag{10}$$

the solution is

$$q = \mathrm{Re}\left[Ae^{i(\omega' + \frac{i}{2}\Gamma)t}\right] \tag{11}$$

where

$$\omega' = \sqrt{\omega_0^2 - \frac{\Gamma^2}{4}} \tag{12}$$

substitute

$$q(t=0) = c \cdot v(3\tau) \tag{13}$$

we get

$$A = 4.75 \cdot 10^{-9} \tag{14}$$

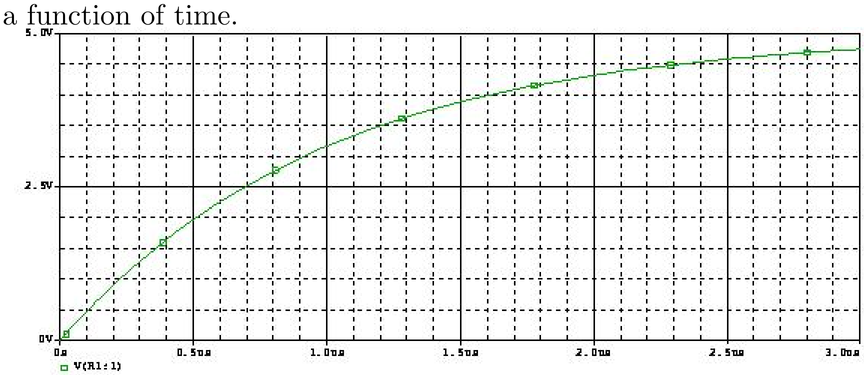
to find the curent:

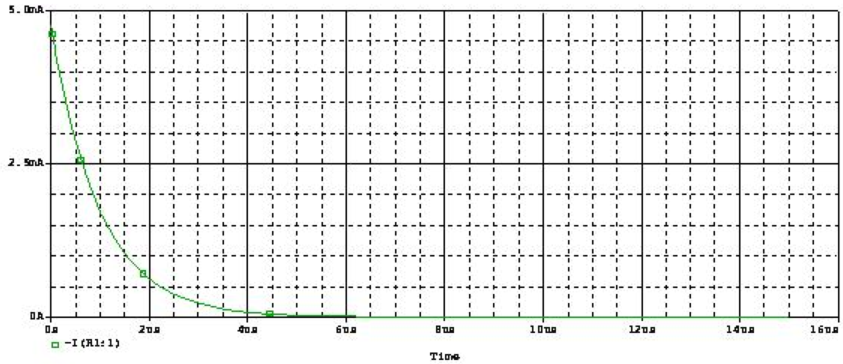
$$I = \dot{q} = RE\left[i(\omega' + \frac{i}{2}\Gamma)Ae^{i(\omega' + \frac{i}{2}\Gamma)t}\right] = -Ae^{-\frac{\Gamma}{2}t} \cdot (\frac{\Gamma}{2}\cos(\omega' t) + \omega' \sin(\omega' t)) \tag{15}$$

eventually substitute the given parameters we get:

$$I = -4.75 \cdot 10^{-9} e^{-37509t}\left[37509 \cdot \cos(8\frac{2}{3} \cdot 10^6 t) + 8\frac{2}{3} \cdot 10^6 \sin(8\frac{2}{3} \cdot 10^6 t)\right]_{Amper} \tag{16}$$

The first plot presents a voltage on the resistor R and on the second one the current through it as a function of time.




5.0mA
2.5mA
0A
0s
2us
4us
6us
8us
10us
12us
14us
16us
-I(R1:1)
Time