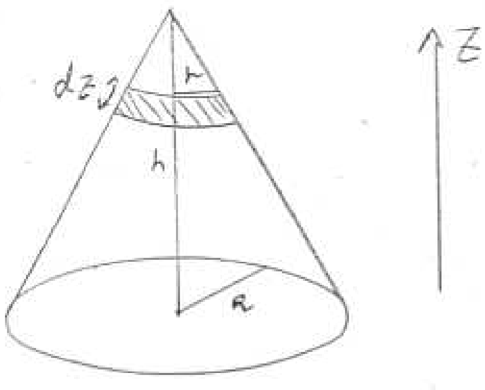

$$dm = \rho\, dV = \rho \pi r^2 dz$$

$$\frac{h}{R} = \frac{h-z}{r}$$ מדמיון משולשים:

$$r = R\frac{h-z}{h}$$

$$dm = \rho \pi r^2 dz = \rho \pi R^2 \frac{(h-z)^2}{h^2} dz$$

$$z_{cm} = \frac{\int z\,dm}{\int dm}$$

$$\int z\,dm = \rho \pi R^2/h^2 \int_0^h z\,dz\,(h-z)^2 = \rho\pi \frac{R^2}{h^2} \int_0^h (zh^2 - 2hz^2 + z^3)\,dz =$$

$$= \rho\pi \frac{R^2}{h^2}\left[\frac{z^2}{2}h^2\Big|_0^h - 2h\frac{z^3}{3}\Big|_0^h + \frac{z^4}{4}\Big|_0^h \right] =$$

$$= \rho\pi \frac{R^2}{h^2}\left[\frac{h^4}{2} - \frac{2}{3}h^4 + \frac{1}{4}h^4\right] = \rho\pi \frac{R^2}{12} h^2$$

$$\int dm = \rho\pi\frac{R^2}{h^2}\int_0^h dz\,(h-z)^2 = \rho\pi\frac{R^2}{h^2}\int_0^h dz\,(h^2 - 2hz + z^2) =$$

$$= \rho\pi\frac{R^2}{h^2}\left[h^2 z\Big|_0^h - 2h\frac{z^2}{2}\Big|_0^h + \frac{z^3}{3}\Big|_0^h\right] = \rho\pi\frac{R^2}{h^2}\left[h^3 - h^3 + \frac{h^3}{3}\right] =$$

$$= \frac{\rho\pi R^2}{3} h$$

$$z_{cm} = \frac{\int z\,dm}{\int dm} = \frac{\rho\pi R^2 \frac{h^2}{12}}{\rho\pi R^2\frac{h}{3}} = \frac{1}{4}h$$