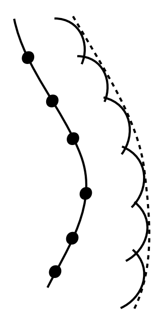
# Chapter 7

# Diffraction

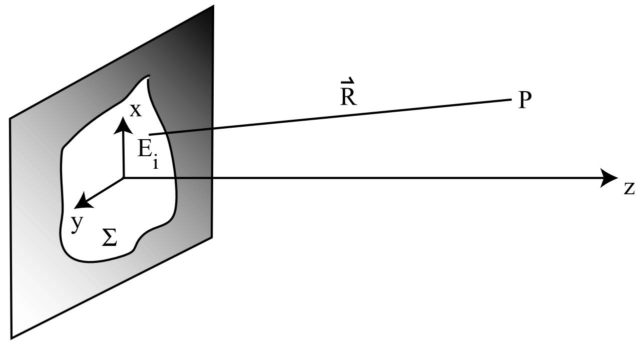
If an opaque object is placed between a point source of light and a white screen, it is found that the shadow that is cast by the object departs from the perfect sharpness predicted by geometrical optics. Close examination of the shadow edge reveals that some light goes over into the dark zone of the geometrical shadow and that dark fringes appear in the illuminated zone. This "smearing" of the shadow edge is closely related to another phenomenon, namely, the spreading of light after passing thorough a very small aperture, such as a pinhole or a narrow slit. The collective name given to these department from geometrical optics is *diffraction*.

## 7.1 Huygen's Principle

The Huygen's principle states that "every point on a wavefront can be considered to be a source of a spherical wave (or Huygen's wavelet), and the envelope of the wavelets gives the wavefront at a later time".



Crudely speaking, the Huygen's Principle includes interference effects in the ray propagating, and allows interference happens while the wave propagates. Huygen's Principle is usually applied to diffraction problems in the following simple way. Suppose we know the (scalar) optical field $E(x, y)$ over some aperture in a plane $\Sigma$. The basic idea is to consider each point in the aperture to be a source of Huygen's wavelets. The total field at an observation point $P$ is just the (integral) sum of all the wavelets.




Roughly speaking, the $j^{th}$ element in the aperture contributes a field $E_i(\vec{r_j})$ to the total field in $\Sigma$, and radiates a Huygen's spherical wave

$$E_i(r_j)\frac{e^{-i\vec{k}\cdot\vec{R}}}{R}$$

Now, again we crudely say that the total field at $P$ is the sum of all the element $j$ in $\Sigma$,

$$E_p = \int\int_{\Sigma} E_i(x,y)\frac{e^{-i\vec{k}\cdot\vec{R}}}{R}dxdy$$

However, this is not the full story of the diffraction theory. The above physical description can gives us an very simple intuitive picture of how diffraction occurs, but it lacks some details in the full diffraction theorem. In fact, a factor $(i/\lambda)$ needs to be included in the above integral. We shall see how this factor comes in to the equation from the rigorous treatment in the next session. But roughly speaking, $i$ comes essentially from the Guoy phase shift and $1/\lambda$ can be considered as "source strength". Thus

$$\boxed{E_p = \frac{i}{\lambda}\int\int_{\Sigma} E_i(x,y)\frac{e^{-i\vec{k}\cdot\vec{R}}}{R}dxdy}$$

This is known as the *Huygen's-Fresnel* integral.

## 7.2 Fresnel-Kirchhoff theorem

Huygen's principle qualitatively says we should be able to find the field at any point $P_0$ given some initially known wave. From vector calculus, we have the divergence theorem as

$$\int\int_S \vec{F}\cdot\hat{n}ds = \int\int\int_V \nabla\cdot\vec{F}dv$$

where $\vec{F}$ is a vector function, and $s$ is the surface enclosing volume $v$. Now, let $\vec{F} = \phi\nabla\psi$, where $\phi$ and $\psi$ are scalar functions. we have

$$\begin{aligned}\int\int_S(\phi\nabla\psi\cdot\hat{n})ds &= \int\int\int_V \nabla\cdot(\phi\nabla\psi)dv \\ &= \int\int\int_V(\phi\nabla^2\psi + \nabla\phi\cdot\nabla\psi)dv\end{aligned}$$

Again, we can let $\vec{F} = \psi \nabla \phi$ and do this again, we have another similar equation

$$\int\int_S (\psi \nabla \phi \cdot \hat{n}) ds = \int\int\int_V (\psi \nabla^2 \phi + \nabla \psi \cdot \nabla \phi) dv$$

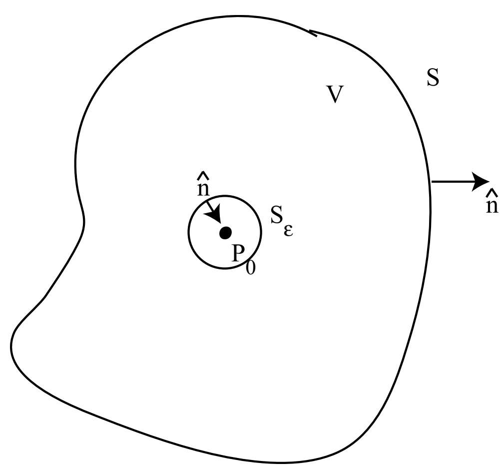
Subtracting the two yields the *Green's theorem* as

$$\int\int_S (\phi \nabla \psi - \psi \nabla \phi) \cdot \hat{n} ds = \int\int\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dv$$

The Green's theorem is often written in a slightly different from, using $\nabla \psi \cdot \hat{n} = \frac{\partial \psi}{\partial n}$, where $\hat{n}$ is the unit vector normal to the surface $s$

$$\boxed{\int\int_S (\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n}) ds = \int\int\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dv}$$

Now, suppose we know $E = \phi(\vec{r}) e^{i\omega t}$ on some surface $S$. Given $\phi$ on $S$, we want to know $\phi$ at some point $P_0$ inside $S$.




where $S_\epsilon$ is a small surface spherical enclosing $P_0$ and $V$ is the volume bounded by $S$ on the outside and $S_\epsilon$ on the inside. Here, we also assume that inside the volume $V$, there are no "electromagnetic sources"; therefore, the Helmholtz (or wave) equation holds inside the volume $V$, and we have,

$$(\nabla^2 + k^2)\psi = 0 \quad , \quad (\nabla^2 + k^2)\phi = 0$$

applying these two equation to the right side of the Green's theorem, we have

$$\int\int\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dv = \int\int\int_V k^2 (\phi \psi - \psi \phi) dv = 0$$

Therefore, for the left side of the Green's theorem,

$$-\int\int_{S_\epsilon} (\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n}) ds = \int\int_S (\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n}) ds$$

Note that the surface normal of $S_\epsilon$ points into $P$ and the surface normal of $S$ points away from $P$; therefore, the integral on the left side of the equation is negative.
The solution $\psi(\vec{r})$ of the wave equation in the spherical coordinate is

$$\psi(\vec{r}) = \frac{e^{-i\vec{k}\cdot\vec{r}}}{r}$$

where $\vec{r}$ is the distance measuring from $P_0$ to the two surfaces.
First lets consider the integral over $S_\epsilon$

$$\int\int_{S_\epsilon}(\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n})ds \quad = \quad \int\int_{S_\epsilon}[\phi\frac{\partial}{\partial n}(\frac{e^{-i\vec{k}\cdot\vec{r}}}{r}) - (\frac{e^{-i\vec{k}\cdot\vec{r}}}{r})\frac{\partial\phi}{\partial n}]ds$$

Since $\hat{r}$ is pointing away from $P_0$ towards the inner surface, and $\hat{n}$ is pointing towards $P_0$ from the inner surface, so $\hat{r}\cdot\hat{n} = -1$,

$$\frac{\partial}{\partial n}(\frac{e^{-i\vec{k}\cdot\vec{r}}}{r}) = \nabla(\frac{e^{-i\vec{k}\cdot\vec{r}}}{r})\cdot\hat{n} = -(\hat{r}\cdot\hat{n})(\frac{1}{r^2} + \frac{ik}{r})e^{-i\vec{k}\cdot\vec{r}} = (\frac{1}{r^2} + \frac{ik}{r})e^{-i\vec{k}\cdot\vec{r}}$$

Since we have integrating the inner surface $S_\epsilon$, $r_{01} = \epsilon$ is the radius of the inner surface, and we can introduce the solid angle $d\Omega = dS/\epsilon^2$. Then,

$$\int\int_{S_\epsilon}[\phi(\frac{1}{\epsilon^2} + \frac{ik}{\epsilon})e^{-ik\epsilon} - (\frac{e^{-ik\epsilon}}{\epsilon})\frac{\partial\phi}{\partial n}]\epsilon^2 d\Omega = \int\int_{S_\epsilon}[(1 + ik\epsilon)\phi - \epsilon\frac{\partial\phi}{\partial n}]e^{-ik\epsilon}d\Omega$$

Now take the limit $\epsilon \to 0$, which means the inner surface $S_\epsilon$ shrinks to an infinitesimal volume around $P_0$, and the only nonzero term in the above integral is 1, so

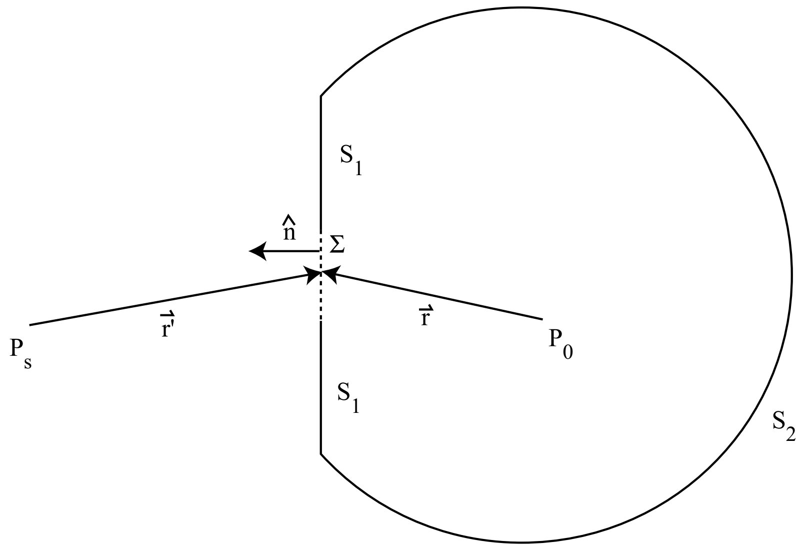
$$\int\int_{S_\epsilon}\phi d\Omega = 4\pi\phi(P_0)$$

Therefore, with the outer surface integral, we have

$$\boxed{\phi(P_0) = \frac{1}{4\pi}\int\int_S[(\frac{e^{-i\vec{k}\cdot\vec{r}}}{r})\frac{\partial\phi}{\partial n} - \phi\frac{\partial}{\partial n}(\frac{e^{-i\vec{k}\cdot\vec{r}}}{r})]ds}$$

This is called the *Helmholtz-Kirchhoff integral theorem*. This theorem means that given that we know the field and its first derivative on any arbitrary surface $S$ enclosing $P_0$, we can calculate the field $\phi(P_0)$.

Now suppose we have a point source at $P_s$, and we want to know the field at $P_0$, but there is an opaque screen containing an aperture between them. Therefore, we take our surface $S$ enclosing $P_0$ to be as shown in the graph below, and we have to integrate the electric field at all three surfaces, i.e. $\int\int_S = \int\int_\Sigma + \int\int_{S_1} + \int\int_{S_2}$.




In order to proceed further we must now make some approximation:

1. Assume $S_2$ can be made far enough from $P_0$ that $\phi$ and $\partial\phi/\partial n$ are zero on $S_2$. This is called the *Sommerfield radiation condition*, and is generally satisfy for spherical waves.

2. Assume that the field $\phi$ and its derivative $\partial\phi/\partial n$ are identical to what they would be without the screen $S_1$.

3. Assume that the screen $S_1$ is perfectly opaque that $\phi = 0$ on the screen $S_1$.

The latter two assumptions are known as *Kirchhoff's boundary conditions*. With these assumptions, the Helmholtz-Kirchhoff integral formula reduces to

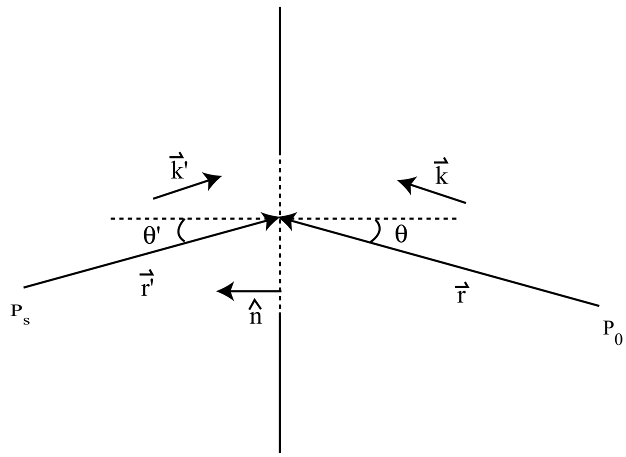
$$\phi(P_0) = \frac{1}{4\pi}\int\int_\Sigma [(\frac{e^{-i\vec{k}\cdot\vec{r}}}{r})\frac{\partial\phi}{\partial n} - \phi\frac{\partial}{\partial n}(\frac{e^{-i\vec{k}\cdot\vec{r}}}{r})]ds$$

Now, if we have a point source at $P_s$, and if the aperture $\Sigma$ is far from $P_s$, then in the aperture,

$$\phi = E_0\frac{e^{-i\vec{k}\cdot\vec{r}'}}{r'}$$

and the first derivative

$$\begin{aligned}\frac{\partial\phi}{\partial n} &= E_0[-ik - \frac{1}{r'}]\frac{e^{-i\vec{k}\cdot\vec{r}'}}{r'}(\hat{r'}\cdot\hat{n}) \\ &\simeq -ik(\hat{r'}\cdot\hat{n})(\frac{e^{-i\vec{k}\cdot\vec{r}'}}{r'})E_0 \quad (r' \gg \frac{1}{k} = \frac{\lambda}{2\pi})\end{aligned}$$




Looking in the aperture region a little more closely, $\hat{r'} \cdot \hat{n} = -\cos\theta'$

$$-ik(\hat{r'} \cdot \hat{n})(\frac{e^{-i\vec{k}\cdot\vec{r}'}}{r'})E_0 = ik\cos\theta'(\frac{e^{-i\vec{k}\cdot\vec{r}'}}{r'})E_0$$

and similarly, $\vec{r} \cdot \hat{n} = \cos\theta$

$$\frac{\partial}{\partial n}(\frac{e^{-i\vec{k}\cdot\vec{r}}}{r}) \simeq -ik(\hat{r} \cdot \hat{n})(\frac{e^{-i\vec{k}\cdot\vec{r}}}{r}) = -ik\cos\theta(\frac{e^{-i\vec{k}\cdot\vec{r}}}{r})$$

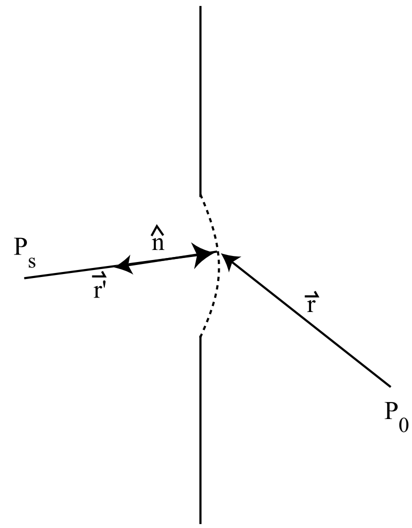
Thus,

$$\phi(P_0) \simeq \frac{E_0}{4\pi}\int\int_{\Sigma}(ik\cos\theta' + ik\cos\theta)\frac{e^{-i\vec{k}\cdot(\vec{r}+\vec{r}')}}{rr'}ds$$

Since $k = 2\pi/\lambda$,

$$\boxed{\phi(P_0) = \frac{i}{\lambda}E_0\int\int_{\Sigma}(\frac{\cos\theta'+\cos\theta}{2})\frac{e^{-i\vec{k}\cdot(\vec{r}+\vec{r}')}}{rr'}ds}$$

This equation called the *Fresnel-Kirchhoff diffraction formula.* It is, in effect, a mathematical statement of Huygen's principle. This is most easily seen by applying the formula to a specific case, namely, that of a circular aperture with the source symmetrically located as shown in the following illustration. The surface of integration is taken to be a spherical cap bounded by the aperture opening.



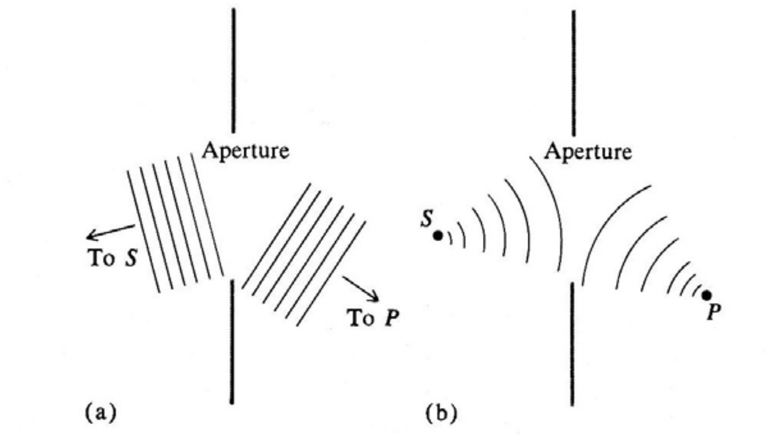

In this case $r'$ is constant and $\theta' = 0 \Rightarrow \cos\theta' = 1$. The Fresnel-Kirchhoff diffraction formula reduces to

$$\phi(P_0) = \frac{i}{\lambda}\int\int_{\Sigma}\phi(\frac{1+\cos\theta}{2})\frac{e^{-i\vec{k}\cdot\vec{r}}}{r}ds$$

where $\phi = E_0\frac{e^{-i\vec{k}\cdot\vec{r}'}}{r'}$. The above equation can be given the following simple interpretation: $\phi$ is the complex amplitude of the incident primary wave at the aperture. From this primary wave each element $ds$ of the aperture gives rise to a secondary spherical wave $\phi\frac{e^{-i\vec{k}\cdot\vec{r}}}{r}ds$. The total optical disturbance at the receiving point $P$ is obtained by summing the secondary waves from each element. However, in the summation it is necessary to take into account the fact $\frac{\cos\theta'+\cos\theta}{2}$, which is known as the *obliquity factor*. In this particular case, $\cos\theta' = 1$, so the obliquity factor is $(1+\cos\theta)/2$. In the forward direction, $\cos\theta \simeq 1$, and the obliquity factor is equal to 1 as well. On the other hand, in the backward direction $\cos\theta \simeq -1$, so the obliquity factor is zero. This explains why there is no backward progressing wave created by the original wavefront. Huygen's principle, as originally proposed, did not include the obliquity factor and thus could not account for the absence of a backward wave. The presence of the factor $-i$ means that the diffracted waves are shifted in phase by 90 degrees with respect to the primary incident wave. This feature was also lacking in the original form of Huygen's principle.

## Fraunhofer and Fresnel diffraction

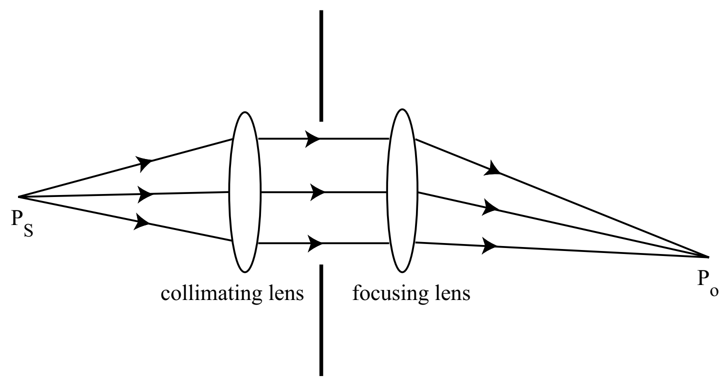
It is generally distinguish diffraction into two different cases. Qualitatively speaking, *Fraunhofer diffraction* occurs when both the incident and diffraction waves are effectively plane waves. This will be the case when the distances from the source to the diffracting aperture and from the aperture to the receiving points are both large enough for the curvatures of the incident and diffracted waves to be neglected. If either the source or the receiving point is close enough to the diffracting aperture so that the curvature of the wavefront is significant, then one has *Fresnel diffraction*.




Diffraction by an aperture. (a) Fraunhofer case; (b) Fresnel case.

## 7.3 Fraunhofer Diffraction

Typically, in order to achieve a planar wavefront to enter and exit the diffraction aperture, a collimating lens and a focusing fens are positioned in between the aperture, as shown in the following graph.




The incident and diffracted wavefronts are therefore strictly plane, and the Fraunhofer case is valid. Several approximation are used for Fraunhofer diffraction:

1. Since the incident and diffracted wavefronts are collimated, and the source is positioned as if at infinite, the aperture is relatively small in this situation; therefore, $\theta \simeq \theta' \simeq 0°$, the obliquity factor $\frac{\cos\theta' + \cos\theta}{2} \simeq 1$.

2. The quantity $\frac{e^{-i\vec{k}\cdot\vec{r}'}}{r'}$ is very nearly constant and can be taken outside the integral.

3. The variation of the remaining factor $\frac{e^{-i\vec{k}\cdot\vec{r}}}{r}$ over the aperture comes principally from the exponential part, so the factor $1/r$ can be replaced by its mean value and taken outside the integral. Also, $\vec{k}$ and $\vec{r}$ are nearly parallel with each other, so $\vec{k}\cdot\vec{r} \simeq kr$.

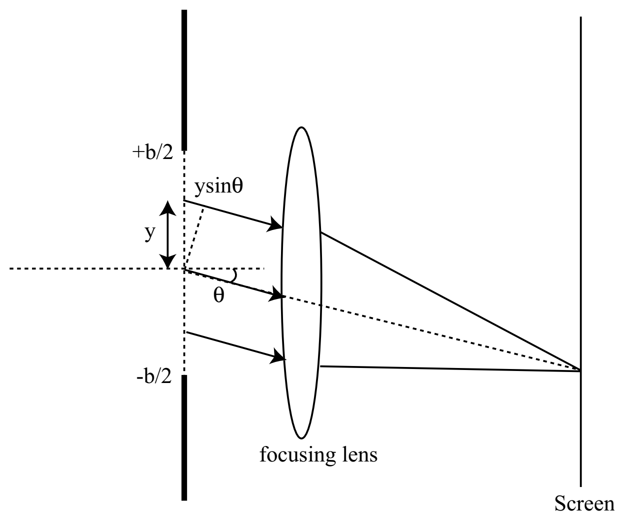
Consequently, the Fresnel-Kirchhoff formula reduces to the very simple equation for Fraunhofer diffraction, and is called the *Fraunhofer diffraction formula*,

$$\boxed{\phi(P_0) = C \int\int_{\Sigma} e^{ikr} ds}$$

where all constant factors have been lumped into one constant $C$. The formula above states that the distribution of the diffracted light is obtained simply by integrating the phase factor $e^{ikr}$over the aperture.

### 7.3.1 Single Slit

The case of diffraction by a single narrow slit is treated here as a one-dimensional problem. Let the slit be of length $L$ and of width $b$. The element of area is then $ds = Ldy$ as indicated in the graph below.




Furthermore, we can express $r$ as

$$r = r_0 + y \sin\theta$$

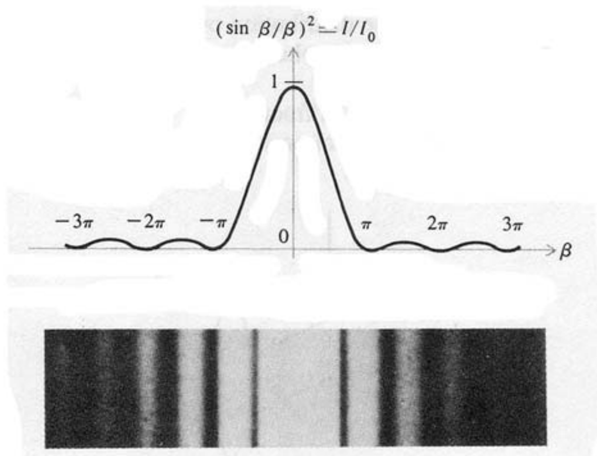
where $r_0$ is the value of $r$ for $y = 0$, and where $\theta$ is the angle shown. The Fraunhofer diffraction formula yields

$$\begin{aligned}
\phi(P_0) &= Ce^{ikr_0} \int_{-b/2}^{+b/2} e^{iky\sin\theta} Ldy \\
&= \frac{CLe^{ikr_0}}{ik\sin\theta} \int_{-b/2}^{+b/2} e^{iky\sin\theta} d(iky\sin\theta) \\
&= CLbe^{ikr_0}[\frac{e^{i(kb\sin\theta/2)} - e^{-i(kb\sin\theta/2)}}{2i}] \cdot [\frac{1}{kb\sin\theta/2}] \\
&= C'(\frac{\sin\beta}{\beta})
\end{aligned}$$

where $\beta = \frac{1}{2}kb\sin\theta$ and $C' = CLbe^{ikr_0}$ is just another constant. Thus $C'(\frac{\sin\beta}{\beta})$ is the total amplitude of the light diffracted in a given direction defined by $\beta$. This light is brought to a focus by the focusing lens, and the corresponding intensity (irradiance) distribution in the focal plane is given by the expression

$$I = I_0(\frac{\sin\beta}{\beta})^2$$

where $I_0 = (CLb)^2$, which is the intensity for $\theta = 0$. The maximum value occurs at $\theta = 0$, and zero values occurs for $\beta = \pm\pi, \pm 2\pi \cdots$.




Fraunhofer diffraction pattern of a single slit.

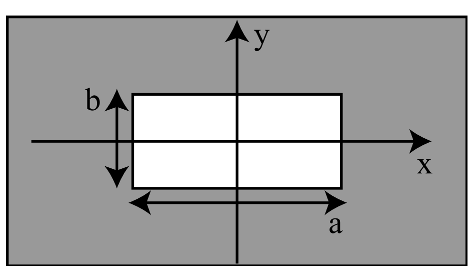
Secondary maxima of rapidly diminishing value occur between these zero values. Thus the diffraction pattern at the focal plane consists of a central bright band, and on either side there are alternating bright and dark bands. The first minimum ($\beta = \pi$) appears at the angle

$$\sin\theta = 2\pi/kb = \lambda/b$$

Therefore, the angular width of the diffraction pattern varies inversely with the slit width, and the amplitude of the central maximum is proportional to the area of the slit. So for very narrow slit (smaller than the optical wavelength), $\theta$ is large and the pattern is dim but wide, and it shrinks and becomes brighter as the slit is widened.

### 7.3.2 Rectangular Aperture

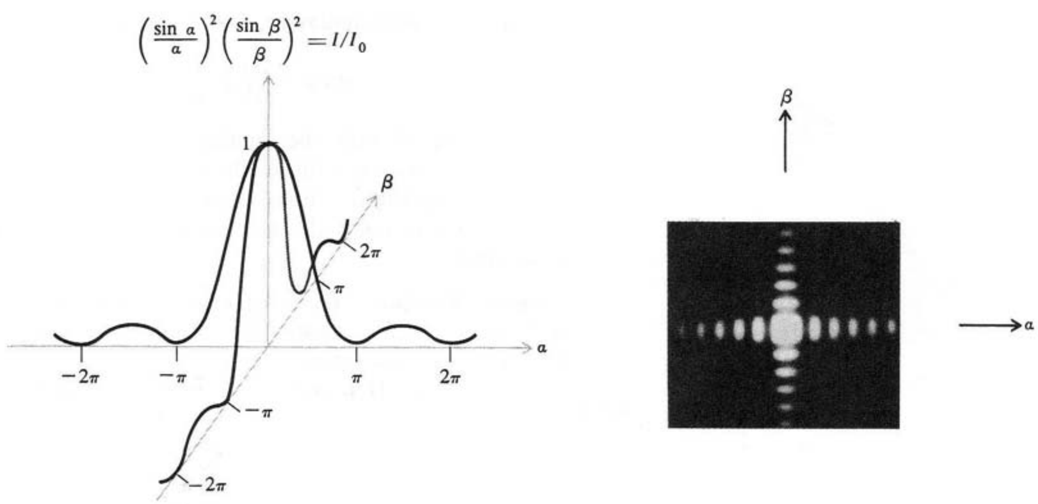
The case of diffraction by a single aperture of rectangular shape is treated in the same way as the single sit, except that one must now integrate in two dimensions.



Therefore, the intensity distribution is given by the product of two single-slit distribution functions in $x$ and $y$ directions; therefore,

$$I = I_0(\frac{\sin\alpha}{\alpha})^2(\frac{\sin\beta}{\beta})^2$$

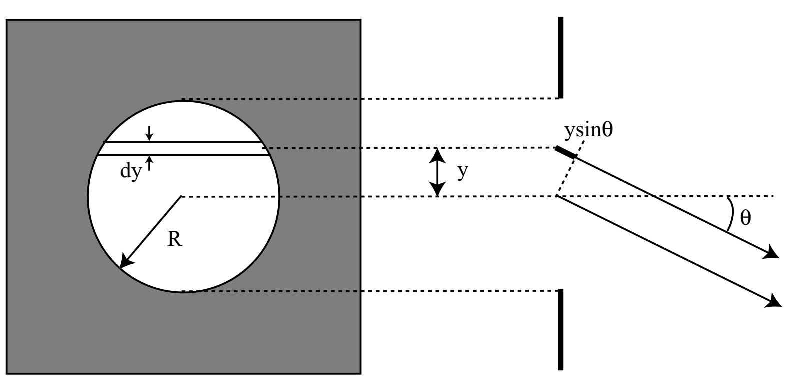
where $\alpha = \frac{1}{2}ka\sin\phi$ and $\beta = \frac{1}{2}kb\sin\theta$. The dimensions of the aperture are $a$ and $b$ and the angle $\phi$ and $\theta$ define the direction of the diffraction ray. The resulting diffraction pattern has lines of zero intensity defined by $\alpha = \pm\pi, \pm 2\pi, \cdots$ and $\beta = \pm\pi, \pm 2\pi, \cdots$. Like the case of a single slit, the scale of the diffraction pattern bears an inverse relationship to the scale of the aperture.




Fraunhofer diffraction pattern of a rectangular aperture.

### 7.3.3 Circular Aperture

To calculate the diffraction pattern of a circular aperture, we choose $y$ as the variable of integration, as in the case of the single slit. If $R$ is the radius of the aperture, then the element of area is taken to be a strip of width $dy$ and length $2\sqrt{R^2 - y^2}$.



The amplitude distribution of the diffraction pattern is then given by

$$\phi(P_0) = Ce^{ikr_0} \int_{-R}^{+R} e^{iky\sin\theta} 2\sqrt{R^2 - y^2} dy$$

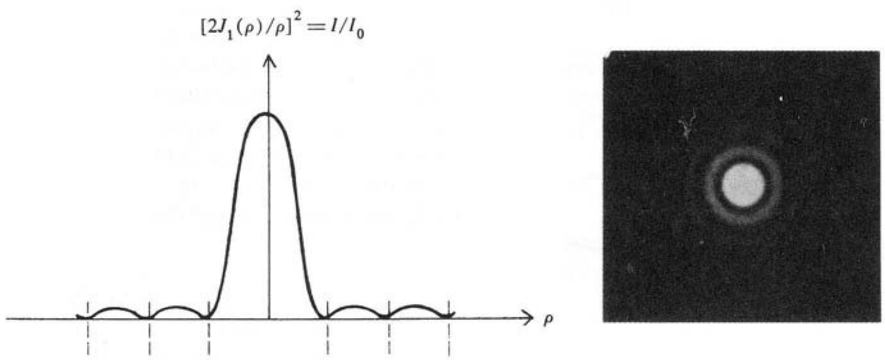
The integral can be simplified by introducing two variables, $u = y/R$ and $\rho = kR\sin\theta$, and the integral becomes

$$\int_{-1}^{1} e^{i\rho u}\sqrt{1-u^2}du = \pi\frac{J_1(\rho)}{\rho}$$

where $J_1(\rho)$ is the first order of the Bessel function of the first kind, and the ratio $J_1(\rho)/\rho \to \frac{1}{2}$ as $\rho \to 0$. Therefore, the intensity distribution of an diffracted circular aperture is given by

$$I = I_0[\frac{2J_1(\rho)}{\rho}]^2$$

where $I_0 = (C\pi R^2)^2$, which is the intensity for $\theta = 0$.




Fraunhofer diffraction pattern of a circular aperture.

The diffraction pattern is circularly symmetric and consists of a bright central disk surrounded by concentric circular bands of rapidly diminishing intensity. The bright central area is known as the *Airy disk*. It extends to the first dark ring whose size is given by the first zero of the Bessel function, namely, $\rho = 3.832$. The angular radius of the first dark ring is thus given by

$$\sin\theta \simeq \theta = \frac{3.832}{kR} = \frac{1.22\lambda}{D}$$

where $D = 2R$ is the diameter of the aperture.

## 7.4 Fresnel Diffraction

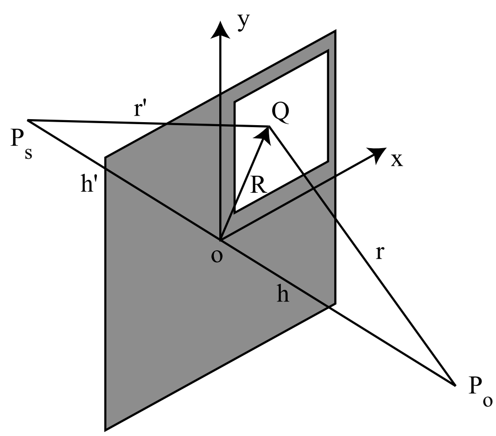
It is considered to be Fresnel diffraction when either the light source or the observing screen, or both, are so close to the diffracting aperture that the curvation of the wavefront becomes significant. Since one is no longer dealing with plane waves, Fresnel diffraction is mathematically more difficult to treat then Fraunhofer diffraction but is actually simpler to observe experimentally because all that is needed is a source of light, an observing screen, and the diffracting aperture. In this section, we will discuss some simple cases of Fresnel diffraction.

### 7.4.1 Rectangular Aperture

Fresnel diffraction by an aperture of rectangular shape is treated by using the Fresnel-Kirchhoff formula.

$$\phi(P_0) = -\frac{i}{\lambda}E_0 \int\int_{\Sigma}(\frac{\cos\theta' + \cos\theta}{2})\frac{e^{-i\vec{k}\cdot(\vec{r}+\vec{r}')}}{rr'}ds$$

We will employ the Cartesian coorindates $x$ and $y$ in the aperture plane as shown in the graph below.




According to the graph, $R^2 = x^2 + y^2$, and

$$\begin{aligned} |\vec{r}+\vec{r}'| &= (h^2+R^2)^{1/2} + (h'+R^2)^{1/2} \\ &= h + h' + \frac{1}{2}R^2(\frac{1}{h}+\frac{1}{h'}) + \cdots \\ &= h + h' + \frac{1}{2L}(x^2+y^2) \end{aligned}$$

where $\frac{1}{L} = (\frac{1}{h}+\frac{1}{h'})$. Again, as in the treatment of Fraunhofer diffraction, we shall assume that the obliquity factor and the radial factor $1/rr'$ vary so slowly compared to the exponential factor $e^{ik(r+r')}$ that they can be taken outside the integral. The Fresnel-Kirchhoff formula then becomes

$$\begin{aligned} \phi(P_0) &= C\int_{x_1}^{x_2}\int_{y1}^{y2} e^{ik(x^2+y^2)/2L}dxdy \\ &= C\int_{x_1}^{x_2} e^{ikx^2/2L}dx \int_{y1}^{y2} e^{iky^2/2L}dy \end{aligned}$$

where $C$ includes all other factors. Upon introducing the dimensionless variables $u$ and $v$ defined as

$$u = x\sqrt{\frac{k}{\pi L}} \qquad v = y\sqrt{\frac{k}{\pi L}}$$

we can rewrite the Fresnel-Kirchhoff formula to be

$$\phi(P_0) = C' \int_{u_1}^{u_2} e^{i\pi u^2/2} du \int_{v_1}^{v_2} e^{i\pi v^2/2} dv$$

where $C' = C\pi L/k$. The above integral can be evaluated as the following,

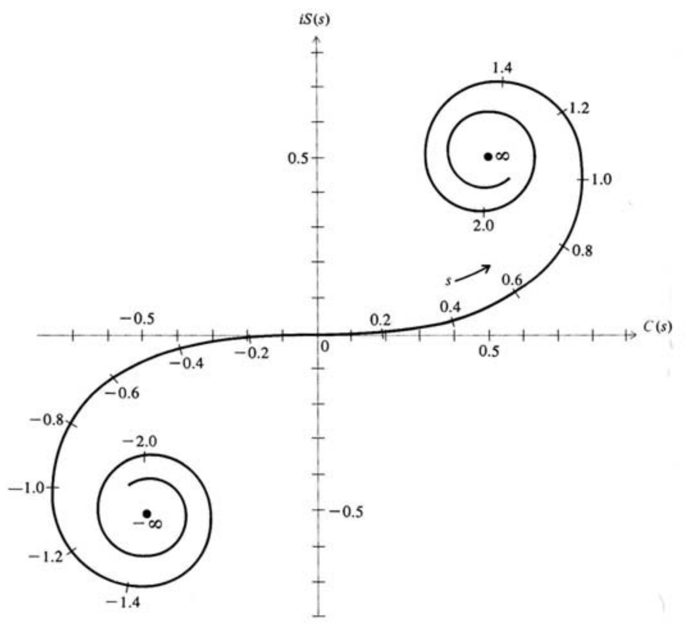
$$\int_0^s e^{i\pi w^2/2} dw = C(s) + iS(s)$$

in which the real and the imaginary parts are given by

$$C(s) = \int_0^s \cos(\pi w^2/2) dw$$

$$S(s) = \int_0^s \sin(\pi w^2/2) dw$$

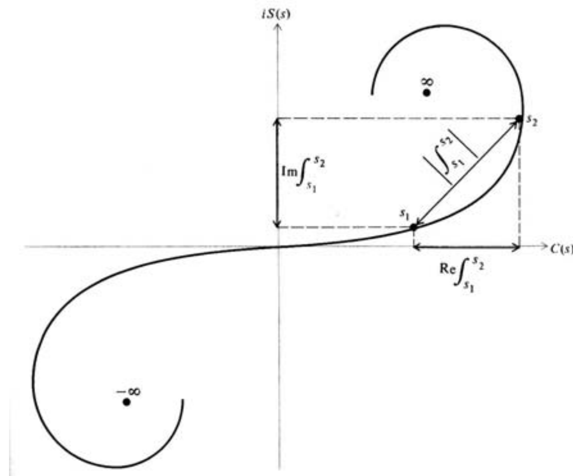
These are known as *Fresnel integrals*. Typically, $C(s)$ is often plotted against $S(s)$, and this curve is called the *Cornu spiral*, as shown in the following graph.




The Cornu spiral is useful for graphical evaluation of the Fresnel integrals. This graph can be used to calculate $\phi(P_0)$, Since

$$\begin{aligned}\int_{s_1}^{s_2} e^{i\pi u^2/2} du &= \int_0^{s_2} e^{i\pi u^2/2} du + \int_{s_1}^{0} e^{i\pi u^2/2} du \\ &= \int_0^{s_2} e^{i\pi u^2/2} du - \int_0^{s_1} e^{i\pi u^2/2} du \\ &= [C(s_2) - C(s_1)] + i[S(s_2) - S(s_1)]\end{aligned}$$

Graphically, mark the limit points $s_1$ and $s_2$ are marked on the spiral. A straight line segment drawn from $s_1$ to $s_2$ then gives the value of the integral $\int_{s_1}^{s_2} e^{i\pi w^2/2}dw$. The length of the line segment is the magnitude of the integral, and the projections on the $C$ and $S$ axes are the real and imaginary parts, respectively.




For illustration purpose, lets consider the case of Fresnel diffraction only by a long slit as a limiting case of a rectangular aperture, by letting $u_1 = -\infty$ and $u_2 = \infty$. This yeilds the formula

$$\phi(P_0) = C'[C(v) + iS(v)]_{v1}^{v2}$$

for the slit where $v_1$ and $v_2$ define the slit edges.
Similarly, for the case of straight-edge, $v_1$ now is also taking the limiting case that $v_1 = -\infty$. This gives

$$\begin{aligned}\phi(P_0) &= C'[C(v) + iS(v)]_{-\infty}^{v2} \\ &= C'[C(v_2) + iS(v_2) + \frac{1}{2} + \frac{1}{2}i]\end{aligned}$$

which is a function of only one variable $v_2$. This variable specifies the position of the diffracting edge. The diffraction pattern is shown in the graph below: