Further Results of General Relativity (GR): a Summary of the most important Results.

וקטורים וטנזורים

1) וקטור קונטרווריאנטי

$$dx^\alpha \to dx'^\alpha = \frac{\partial x'^\alpha}{\partial x^\beta}\, dx^\beta$$

$$A^\alpha \to A'^\alpha = \frac{\partial x'^\alpha}{\partial x^\beta}\, A^\beta$$

2) וקטור קווריאנטי $\left(\phi'(x') = \phi(x)\right)$

$$\frac{\partial \phi}{\partial x^\alpha} \to \frac{\partial \phi}{\partial x'^\alpha} = \frac{\partial \phi}{\partial x^\beta}\frac{\partial x^\beta}{\partial x'^\alpha}$$

$$A_\alpha \to A'_\alpha = \frac{\partial x^\beta}{\partial x'^\alpha}\, A_\beta$$

טנזורים

$$T^\alpha{}_\beta \to T'^\alpha{}_\beta = \frac{\partial x'^\alpha}{\partial x^\gamma}\frac{\partial x^\delta}{\partial x'^\beta}\, T^\gamma{}_\delta$$

3) נגזרת קווריאנטי

$$\frac{\partial}{\partial x^\alpha} V_\beta \to \frac{\partial}{\partial x'^\alpha}\left(\frac{\partial x^\gamma}{\partial x'^\beta} V_\gamma\right)$$

$$\neq \frac{\partial x^\gamma}{\partial x'^\alpha}\frac{\partial x^\delta}{\partial x'^\beta}\frac{\partial V_\delta}{\partial x^\gamma}$$

$$\frac{\partial}{\partial x^\alpha} V^\beta \to \frac{\partial}{\partial x'^\alpha}\left(\frac{\partial x'^\beta}{\partial x^\gamma} V^\gamma\right)$$

$$\neq \frac{\partial x^\kappa}{\partial x'^\alpha}\frac{\partial x'^\beta}{\partial x^\theta}\frac{\partial V^\theta}{\partial x^\kappa}$$

נגזרת קווריאנטי (2

$$V^{\mu}{}_{;\lambda} \equiv \frac{\partial V^{\mu}}{\partial x^{\lambda}} + \Gamma^{\mu}_{\lambda\kappa} V^{\kappa}$$

$$V'^{\mu}{}_{;\lambda} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\rho}}{\partial x'^{\lambda}} V^{\nu}{}_{;\rho}$$

$$V_{\mu;\nu} \equiv \frac{\partial V_{\mu}}{\partial x^{\nu}} - \Gamma^{\lambda}_{\mu\nu} V_{\lambda}$$

$$V'_{\mu;\nu} = \frac{\partial x^{\rho}}{\partial x'^{\mu}} \frac{\partial x^{\sigma}}{\partial x'^{\nu}} V_{\rho;\sigma}$$

$$\Gamma^{\mu}_{\lambda\kappa} = \left\{ {\mu \atop \lambda\kappa} \right\}$$

$$T^{\mu\sigma}{}_{\lambda;\rho} = \frac{\partial T^{\mu\sigma}{}_{\lambda}}{\partial x^{\rho}} + \Gamma^{\mu}_{\rho\nu} T^{\nu\sigma}{}_{\lambda} + \Gamma^{\sigma}_{\rho\nu} T^{\mu\nu}{}_{\lambda} - \Gamma^{\kappa}_{\lambda\rho} T^{\mu\sigma}{}_{\kappa}$$

$$g_{\mu\nu;\lambda} = \frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} - \Gamma^{\rho}_{\lambda\mu} g_{\rho\nu} - \Gamma^{\rho}_{\lambda\nu} g_{\rho\mu} = 0.$$

1) קבועים α, β

$$(\alpha A^{\mu}{}_{\nu} + \beta B^{\mu}{}_{\nu})_{;\lambda} = \alpha A^{\mu}{}_{\nu;\lambda} + \beta B^{\mu}{}_{\nu;\lambda}$$

2) $(A^{\mu}{}_{\nu} B^{\lambda})_{;\rho} = A^{\mu}{}_{\nu;\rho} B^{\lambda} + A^{\mu}{}_{\nu} B^{\lambda}{}_{;\rho}$

3) $g_{\mu\nu;\rho} = 0 \quad ; \quad g^{\mu\nu}{}_{;\rho} = 0$

$$(g^{\mu\nu} V_{\nu})_{;\lambda} = g^{\mu\nu} V_{\nu;\lambda}$$

Riemann Christoffel טנזור

$$R^{\lambda}{}_{\mu\nu\kappa} \equiv \frac{\partial \Gamma^{\lambda}_{\mu\nu}}{\partial x^{\kappa}} - \frac{\partial \Gamma^{\lambda}_{\mu\kappa}}{\partial x^{\nu}} + \Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\kappa\eta} - \Gamma^{\eta}_{\mu\kappa}\Gamma^{\lambda}_{\nu\eta}$$

$$R'^{\tau}{}_{\rho\sigma\eta} = \frac{\partial x'^{\tau}}{\partial x^{\lambda}} \frac{\partial x^{\mu}}{\partial x'^{\rho}} \frac{\partial x^{\nu}}{\partial x'^{\sigma}} \frac{\partial x^{\kappa}}{\partial x'^{\eta}} R^{\lambda}{}_{\mu\nu\kappa}$$

טנזור אנרגיה תנע

$$T^{\mu\nu} : \quad T^{\mu\nu}{}_{;\nu} = 0.$$

Einstein טנזור

$$\underbrace{R_{\mu\nu} \equiv R^{\lambda}{}_{\mu\lambda\nu}}, \quad R = g^{\mu\nu} R_{\mu\nu}$$

Ricci טנזור

Einstein טנזור

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R$$

$$G^{\mu\nu} = R^{\mu\nu} - \tfrac{1}{2} g^{\mu\nu} R$$

$G^{\mu\nu}{}_{;\nu} \equiv 0$, and $T^{\mu\nu}{}_{;\nu} = 0$ is the Covariant conservation of energy-momentum ~~the~~ tensor.

Einstein משוואות

$$G_{\alpha\beta} = -8\pi G\, T_{\alpha\beta}$$

both sides have zero divergence $G^{\alpha\beta}{}_{;\beta} = T^{\alpha\beta}{}_{;\beta} = 0$

if $g_{00} \approx 1 + 2\phi$, $g_{ij} \approx -\delta_{ij}$

$$\Longrightarrow \quad \nabla^2 g_{00} = -8\pi G\, T_{00}$$

$(\rho = T_{00})$ $\quad \nabla^2 \phi = -4\pi G \rho$

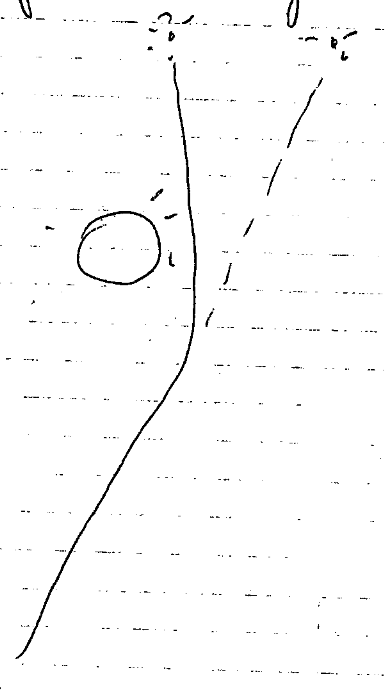
The Schwarzschild solution of empty Einstein's equations (<u>exact solution</u>)

$$g_{00} = \left(1-\frac{2GM}{r}\right), \quad g_{rr} = -\left(1-\frac{2GM}{r}\right)^{-1}$$

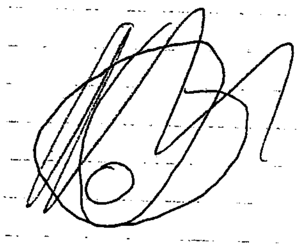
$$g_{\theta\theta} = -r^2, \quad g_{\phi\phi} = -r^2\sin^2\theta$$

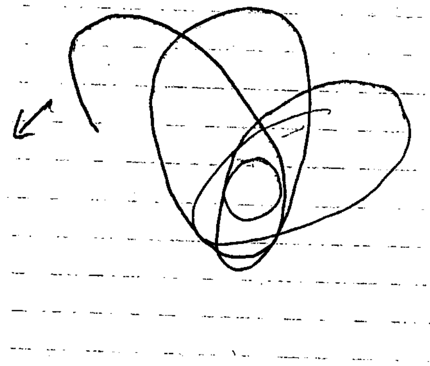
1 Classical Tests: 1) Red shift (studied ✓)

2) Deflection of light



3) Prescession Perihelion





Other Subjects to be studied.
(may be).

Cosmology & Expansion of the Universe, Big Bang theory, Inflation theory
Observations of Accelerated Universe.

$$ds^2 = dt^2 \cancel{A} - R(t)^2 (dx^2 + dy^2 + dz^2)$$

Geodesics "comoving observers" are for example $x = x_0$ = קבוע

$y = y_0$

$z = z_0$

Distance between observers ("Galaxies").

$$\Delta l = R(t)\,|\Delta\vec{x}|$$

$$|\Delta\vec{x}| = \text{קבוע}$$

Gravitational Waves.

Action Principle & Symmetries.

Other Theories of Gravity.