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ABSORPTION OF RADIATION BY SOLAR COLLECTORS

The primary mechanism in the chain leading from solar energy in space to one of the more familiar forms in which this energy manifests itself on earth, is absorption of radiation by a material body. The latter may be a living plant in which a chemical reaction is initiated by the incoming radiation, a photovoltaic cell in which the radiation produces an electric current, or merely a blackened surface which produces heat. We shall take the absorption of radiation by a surface as our starting point.

Black bodies

The engineering literature defines *monochromatic emissive power* $E_{b\lambda}(\lambda,T)$ of a black body radiator via *Planck's Law:*

$$E_{b\lambda}(\lambda,T) = \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)} \tag{4.1}$$

Where the two constants C_1 and C_2 are defined in terms of a number of fundamental constants of physics, specifically:

$$C_1 = 2\pi c^2 h = 3.74 \times 10^8 \text{ W } \mu\text{m}^4 \text{ m}^{-2} \tag{4.2}$$

$$C_2 = ch/k = 1.44 \times 10^4 \ \mu\text{m K} \tag{4.3}$$

where $c = 2.998 \times 10^8$ m s^{-1} is the velocity of light, $h = 6.626 \times 10^{-34}$ J s is Planck's constant, and $k = 1.381 \times 10^{-23}$ J K^{-1} is Boltzmann's constant. [Note the mixed units employed for distances: meters and microns. This is convenient since wavelengths are usually expressed in the smaller of the two units].

A useful relation, *Wien's Displacement Law,* may be obtained from the first derivative of Planck's Law. It shows that a reciprocal relationship exists between the maximum value of $E_{b\lambda}(\lambda,T)$ and the temperature:

$$\lambda_{max} T = 2898 \ \mu\text{m K} \tag{4.4}$$

For example, Wien's Law enables us to rapidly determine that the spectrum emitted by a black object at the temperature of the sun's effective surface temperature, T = 5770 K, peaks at a wavelength of 0.50 µm, or that the spectrum of radiation emitted by a soot-covered pot of boiling water will peak at 7.8 µm, etc.

For any given temperature T, eq (**4.1**) expresses the power radiated by a black body as a function of wavelength. This expression may be integrated over all wavelengths to give the *total emissive power* E_b of the black body:

$$E_b = \int_0^\infty E_{b\lambda}(\lambda,T)\, d\lambda = \sigma T^4$$

(4.5)

where $\sigma = (2\pi^5 k^4) / (15 c^2 h^3) = 5.671 \times 10^{-8}$ W m^{-2} K^{-4} is called the Stefan-Boltzmann constant. An analytical method of solving the integral in eq (**4.5**) can be found in [**1**]. Eq (**4.5**), the *Stefan-Boltzmann Law,* is the basic formula for the exchange of radiation between surfaces at different temperatures.

One often needs to perform integrals similar to eq (**4.5**) but for which the upper limit of integration is finite. For such purposes we may note from eq (**4.1**) that $E_{b\lambda}(\lambda,T) / \sigma T^5$ is a function of the product λT only. Thus the quantity:

$$F_{0\to\lambda}(\lambda T) \equiv \frac{E_{0\to\lambda}(\lambda,T)}{\sigma T^4} = \int_0^{\lambda T} d(\lambda T)\, \frac{E_{b\lambda}(\lambda,T)}{\sigma T^5}$$

(4.6)

represents the *fractional power* in the wavelength interval $0 \le \lambda$. These integrals must be performed numerically but they are tabulated, for various values of λT, in many textbooks on heat transfer, e.g. [**2**], and in the appendix to this lecture.

Example 1: Which fractions of the total power radiated by a black body at 5770 K fall, respectively, in the UV ($0 \le \lambda \le 0.38$ µm), the visible (0.38 µm $\le \lambda \le$ 0.7 µm) and the IR (0.7 µm $\le \lambda \le \infty$) parts of its spectrum?

Solution: For the UV part of the spectrum, λT takes the value 2193µm K at the upper limit of integration. The tabulated value of $F_{0 \to \lambda}(\lambda T)$ under these conditions is 0.10. Hence 10% of the radiation is ultraviolet. At the upper part of the visible spectrum λT takes the value

4039μm K for which $F_{0 \to \lambda}(\lambda T)$ is found to be 0.48. Hence 48% of the radiation is ultraviolet plus visible. Subtracting the former result we infer that 38% is visible. Finally, from the overall normalization of unity, the remaining, infrared, part must comprise 52% of the total. These percentages are quite similar to those measured in the actual solar spectrum above the earth's atmosphere (See **Lecture 1**).

Example 2: Silica glass transmits 92% of incident radiation in the wavelength range 0.35 μm - 2.7 μm, but is opaque at other wavelengths. What percentage of solar radiation will the glass transmit?

Solution: We approximate the solar spectrum as that of a black body at 5770 K. The fraction of incident energy at all wavelengths up to 2.7 μm is thus given by $F_{0 \to 2.7\ \mu m}(15579)$ = 0.97. The fraction at all wavelengths up to 0.35 μm is similarly found to be $F_{0 \to 0.35\ \mu m}(2020)$ = 0.07. Therefore the total fraction of incident energy in the transmittance range of the glass is 0.90. Hence the total percentage of solar radiation transmitted = 0.90 x 92% = 83%.

Non-black bodies

All physical objects absorb and emit electromagnetic radiation but they do not necessarily radiate as black bodies. For a non-black body it is convenient to define a *hemispherical monochromatic emissivity* $\varepsilon_\lambda(\lambda,T)$ as the ratio of its emissive power at a given wavelength, to that of a black body at the same wavelength:

$$\varepsilon_\lambda(\lambda,T) = E_\lambda(\lambda,T) / E_{b\lambda}(\lambda,T) \qquad \textbf{(4.7)}$$

The fact that $E_\lambda(\lambda,T)$ or $\varepsilon_\lambda(\lambda,T)$, for a given surface, is in general different at different wavelengths gives the surface the characteristics of a *selective emitter* - a property that may be tailored with important technological consequences, as will be seen below. If eq (**4.7**) is integrated over all wavelengths we obtain, using eq. (**4.5**), the *total emissivity* of that surface:

$$\varepsilon(T) \equiv \frac{E(T)}{\sigma T^4} = \int_0^\infty \frac{\varepsilon_\lambda(\lambda,T)\, E_{b\lambda}(\lambda,T)}{\sigma T^4}\, d\lambda \qquad \textbf{(4.8)}$$

where E(T) is called the *emissive power* of the non-black surface at temperature T. The emissive power represents the radiant flux from a surface due solely to its temperature. It is therefore an intrinsic property of the surface.

Example 3: The absorbing surface of a hypothetical solar collector exhibits a (temperature independent) monochromatic emissivity of $\varepsilon_\lambda(\lambda)$ = 0.9 for wavelengths up to 2.5 µm and $\varepsilon_\lambda(\lambda)$ = 0.1 for wavelengths above 2.5 µm [**Fig 1**]. What is its total emissivity for operation at (a) 150 ^{o}C, (b) 1500 ^{o}C ?

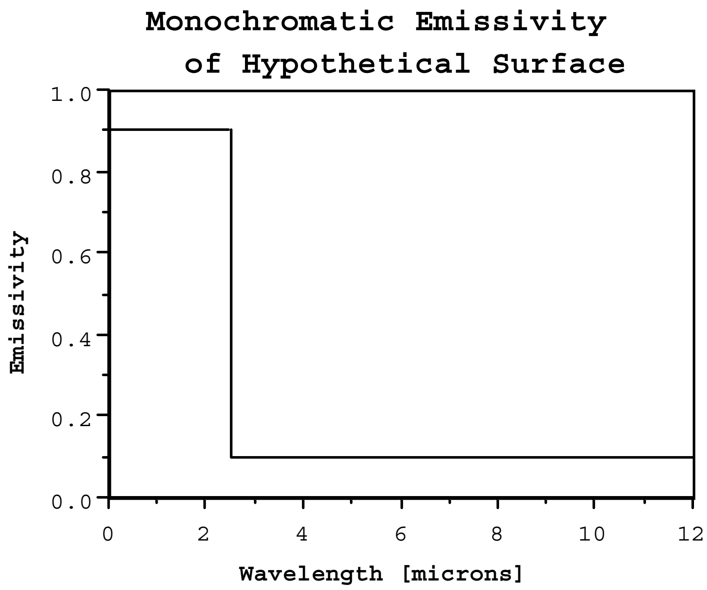

Figure 1: Monochromatic emissivity $\varepsilon_\lambda(\lambda)$ for a hypothetical solar collector absorber plate

Solution: From eq (**4.8**) we obtain:

$$\varepsilon_\lambda(\lambda,T) \;=\; (0.9)\, F_{0 \to 2.5\ \mu m}(\lambda T) + (0.1)\,[1 - F_{0 \to 2.5\ \mu m}(\lambda T)] \qquad \textbf{(4.9)}$$

(a) At T = 423 K, λT = 1058 $_{\mu m}$K, and $F_{0 \to 2.5\ \mu m}(\lambda T) \approx 0$. Hence ε = 0.10

(b) At T = 1773 K, λT = 4433 $_{\mu m}$K, and $F_{0 \to 2.5\ \mu m}(\lambda T) \approx 0.55$. Hence ε = 0.54

A surface may also radiate power as a result of the absorption and re-radiation of incoming radiation. The term *irradiation* is used to denote the flux of incoming radiation from an external source. One refers to monochromatic irradiation G_λ, or to total irradiation G equal to G_λ integrated over all wavelengths.

In such circumstances one needs to know the *monochromatic absorptivity* α_λ of the surface. This is defined as the fraction of monochromatic irradiance that is absorbed. Clearly, α_λ is not an intrinsic property of the surface as it might depend on the direction of the incoming radiation. However, simple thermodynamic arguments may be used to prove that for any given direction:

$$\alpha_\lambda = \varepsilon_\lambda \qquad \textbf{(4.10)}$$

Moreover, if the surface is a so-called *diffuse* absorber - i.e. one without directional characteristics then eq (**4.10**) is true for all absorbed and emitted radiation at the wavelength λ, irrespective of its direction. Finally, if the surface exhibits no wavelength selective properties then:

$$\alpha = \varepsilon \qquad \textbf{(4.11)}$$

- a result similar in form to the famous *Kirchhoff's Law* in statistical mechanics (see, for example, **[1]**), although in that situation (unlike the present case) the emitting surface and the source are supposed to be in thermal equilibrium.

Example 4: What is the total absorptivity of the surface in Example 3, for radiation received from a black body at 5770 K ?

Solution: $\alpha_\lambda(\lambda,T) = \varepsilon_\lambda(\lambda,T)$ [Our hypothetical surface is assumed to be a diffuse absorber]. We evaluate eq. (**4.9**) again, but at a temperature of 5770 K. We now have $\lambda T = 14{,}425$ µm K and $F_{0 \rightarrow 2.5\,\mu m}(\lambda T) \approx 0.97$. Hence $\alpha = 0.88$.

In similar manner, a *monochromatic reflectivity* ρ_λ and *monochromatic transmissivity* τ_λ also characterize the spectral response of non-black bodies to incoming radiation. They represent the respective fractions of incoming radiation at a given wavelength that are reflected and transmitted by the surface. Their integrals over all wavelengths are correspondingly denoted by ρ and τ. By energy conservation we must clearly have:

$$\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1 \qquad \textbf{(4.12)}$$

and

$$\alpha + \rho + \tau = 1 \qquad \textbf{(4.13)}$$

Clearly, like α, both ρ and τ depend upon the spectral distribution of the source and are not, therefore, intrinsic properties of a surface.

Selective surfaces

Fig 2 displays $E_{b\lambda}(\lambda,T) / \sigma T^4$, normalized to 1360 W/m^2, for black bodies at 5762 K and 373 K. These curves approximate the so-called AM0 *(Air Mass zero)* solar spectrum and that of a black body at 100^{o}C, respectively. We notice that there is an almost complete separation between the two spectra. If we could construct a hypothetical surface that were perfectly absorbing in the range $0 \leq \lambda \leq 2.5$ μm and perfectly reflecting in the range 2.5 μm $\leq \lambda \leq \infty$, the surface would constitute an ideal solar collector for such low temperature purposes. That is to say, it would absorb practically all of the incoming solar radiation but emit no far infra-red energy.

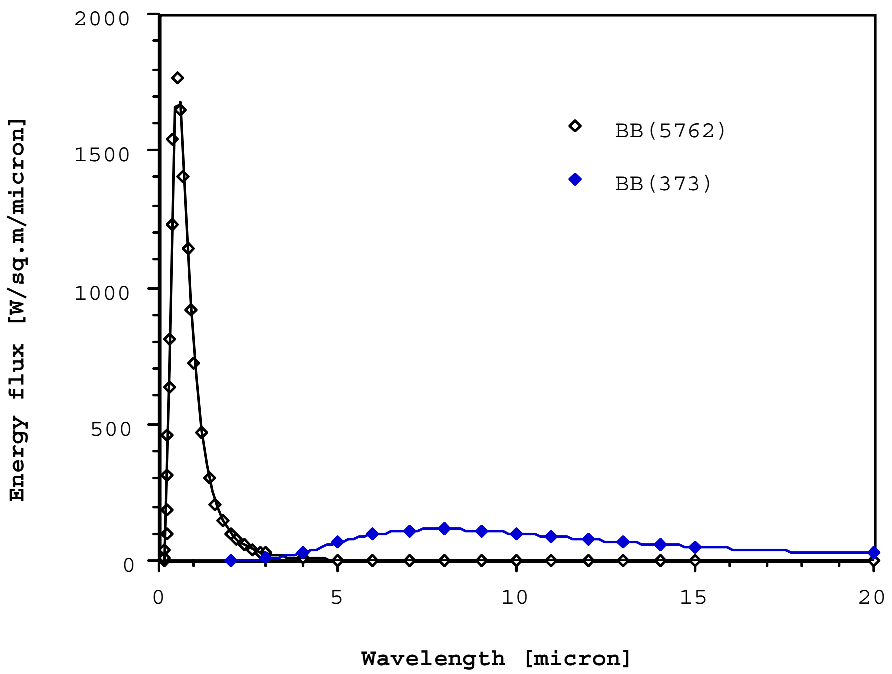

Figure 2: Black body spectra for 5762 K and 373 K

The hypothetical surface considered in Examples 3 and 4 above is similar to the kind of surface properties envisioned here, but in order to give a slightly more realistic picture we considered absorptivities of 0.9 and 0.1 rather than 1 and 0 for the two respective wavelength ranges. From the results of those calculations such a surface would evidently exhibit desirable selective properties for low temperature use but not at higher temperatures.

In fact there do exist a number of so-called *selective surfaces* that approximate such behavior. These surfaces are employed in several commercial solar collectors of the type used for domestic water heating. One example consists of a thin coating of black chrome on nickel. Its spectral emissivity curve is sketched in **Fig. 3**. For higher temperature applications the overlap between the absorption and re-emission spectra of the surface becomes significant. In order to produce a selective surface for such situations it is necessary to fabricate a surface having an absorptivity that approximates a step-function with its step at a wavelength that must be carefully chosen so as to maximize the total power retained by the collector.

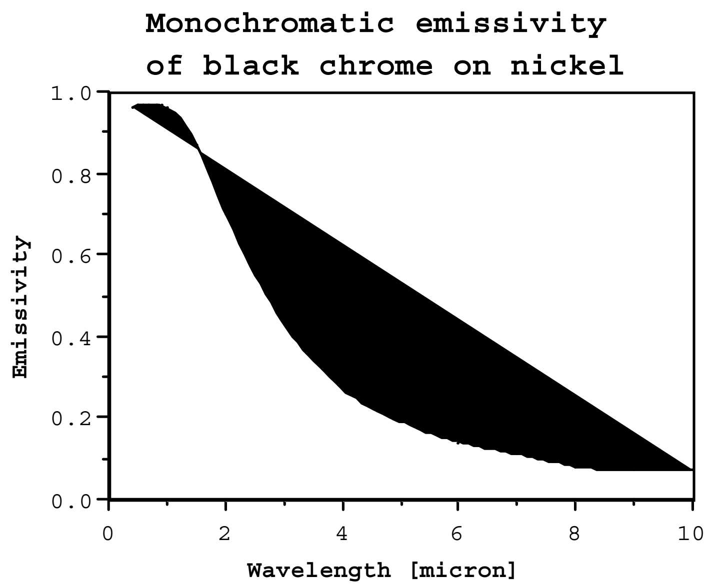

Figure 3: Experimentally observed monochromatic emissivity of black chrome on nickel (Redrawn from [**3**]).

A number of physical processes may be employed to form selective surfaces. Basically the substrate is first given an IR-reflecting coating. This latter is then covered with an IR-transparent material chosen to be highly absorptive for the incoming solar spectrum. Semiconductors constitute an attractive kind of material for the covering since they are naturally transparent to IR and they absorb radiation with wavelengths less than hc/E_g (where E_g is the band gap energy). Reference [**4**] discusses the manner in which selective surfaces, employing semiconductors, may theoretically be tailored for high temperature applications.

Directional aspects of radiant energy

Consider a surface that radiates an amount of flux df into a solid angle $d\omega$ at some angle θ to its normal. Then the quantity

$$df = I_e(\theta,\phi) \cos\theta \; d\omega = I_e(\theta,\phi) \cos\theta \; da / r^2 \qquad \textbf{(4.14)}$$

defines the *intensity* $I_e(\theta,\phi)$ of the emitted radiation. In eq (**4.14**) da is an element of the receiving surface area located at a distance r from the emitting surface but normal to the direction of df. If $I_e(\theta,\phi)$ does not depend on angles the source is said to be a *diffuse emitter* and the integral of the flux over the entire hemisphere at r is simply πI_e. For diffuse radiation the dependence of flux on $\cos\theta$ and r^2, as given in eq. (**4.14**), is known as *Lambert's Law.*

Clearly, the hemispherical integral:

$$\int_h I_e(\theta,\varphi)\cos\theta \, d\omega \qquad \textbf{(4.15)}$$

is equal to the hemispherical emissive power E, in cases when the radiation originates at the emitting surface.

We may similarly consider the flux received *by* an irradiated surface. If $I_i(\theta,\phi)$ denotes the incoming flux from a solid angle element $d\omega$ at an angle θ with the normal to the receiving surface, then an integral similar to (**4.15**) will represent the irradiation G.

Example 5: Calculate the effective black body temperature of the sun in terms of the solar luminosity $L_s = 3.846 \times 10^{26}$ W and the solar radius $R_s = 6.961 \times 10^8$ m.

Solution: If I_s denotes the solar intensity and $r_{s\text{->}e}$ is the sun-to-earth mean distance, the irradiation falling on unit area of receiver is:

$$G = I_s \cos\theta \; \Delta\omega = I_s \cos\theta \,(\pi R_s^2 / r_{s\text{->}e}^2) \qquad \textbf{(4.16)}$$

In eq. (**4.16**) the solar intensity is assumed to be constant over the relatively small solid angle $\Delta\omega = \pi R_s^2 / r_{s\text{->}e}^2$ subtended by the solar disk.

If we approximate the solar intensity as being that due to a black body at temperature T_s, then $I_s = E_b(T_s)/\pi$, where $E_b(T_s)$ is the equivalent black body total emissive power.

For simplicity, we consider our receiving surface as being above the Earth's atmosphere and directed towards the sun so that $\cos\theta = 1$. Then:

$$G = L_s/(4\pi\, r_{s\text{->}e}^2) = (\sigma T_s^4 / \pi) \times (\pi R_s^2 / r_{s\text{->}e}^2) \qquad \textbf{(4.17)}$$

Hence:

$$T_s = \left[\frac{L_s}{4\pi\sigma R_s^2} \right]^{1/4} = 5770\ K$$

(4.18)

Shape factors

In many situations we need to consider the exchange of radiation between objects having arbitrary geometrical forms. For such purposes it is usual to define a so-called *shape factor* $F_{i\text{->}j}$, defined as the fraction of diffuse radiation leaving surface i that reaches surface j.

If A_1 is the area of a black surface 1, the amount of radiation from it that reaches a black surface 2 is:

$$q_{1\text{->}2} = E_{b1}\, A_1\, F_{1\text{->}2} \qquad \textbf{(4.19)}$$

Similarly, the amount of radiation reaching surface 1 from surface 2 is:

$$Q_{2\text{->}1} = E_{b2}\, A_2\, F_{2\text{->}1} \qquad \textbf{(4.20)}$$

The difference between eqs. (**4.19**) and (**4.20**) is the net radiation exchanged between the two surfaces. If both surfaces are at the same temperature, $E_{b1} = E_{b2}$ and hence:

$$A_1\, F_{1\text{->}2} = A_2\, F_{2\text{->}1} \qquad \textbf{(4.21)}$$

Because of this *reciprocity relation* one need consider the surface area of only one of the two radiation-exchanging objects. i.e.

$$(q_{1\text{->}2} - q_{2\text{->}1}) = A_1\, F_{1\text{->}2}\, (E_{b1} - E_{b2}) = A_2\, F_{2\text{->}1}\, (E_{b1} - E_{b2}) \qquad \textbf{(4.22)}$$

In order to obtain the shape factor for a given pair of real surfaces we consider the exchange between two differential areas dA_1 and dA_2 separated by a distance r. Using eq. (**4.14**):

$$\frac{dq_{1\rightarrow 2}}{dA_1} = \frac{E_{b1}}{\pi} \cos\theta_1 \frac{dA_2 \cos\theta_2}{r^2}$$

(4.23)

Hence:

$$dq_{1\rightarrow 2} = E_{b1}\, dA_1 \left[\frac{\cos\theta_1 \cos\theta_2\, dA_2}{\pi r^2} \right]$$

(4.24)

with a corresponding expression for $dq_{2->1}$. The radiation exchanged between the two entire surfaces is then:

$$(dq_{1\rightarrow 2} - dq_{2\rightarrow 1}) = (E_{b1} - E_{b2}) \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{\pi r^2}$$

(4.25)

Comparing this with eq. (**4.22**) we arrive at the general expression for the shape factor:

$$\mathcal{F}_{1\rightarrow 2} = \frac{1}{A_1} \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{\pi r^2}$$

(4.26)

For simple geometries (parallel disks, etc) these shape factor integrals are tabulated in engineering texts. But in many practical situations they have to be performed numerically.

Radiation exchange between parallel planes

For many practical purposes we need to be able to compute the radiation exchange between two parallel planes. These may be the absorber plate and the glazing of a flat plate collector or, equivalently, an inner absorbing cylinder surrounded by a glass tube. In both cases the absorber plate and glass may be considered, in first approximation, as being so-called *gray* emitters or absorbers, i.e. having emissivities ε_1 and ε_2 that are not spectrally dependent. In general the spacing between the planes will be sufficiently small compared with their areas that the latter may be regarded as being essentially infinite in extent. The shape factor $F_{1->2}$ is then, obviously, unity.

If the radiation emitted by surface 1 is $\varepsilon_1\ A_1\ \sigma T^4\ A_1\ \sigma T^4$, then the amount absorbed by surface 2 will be $\varepsilon_2\ \varepsilon_1\ A_1\ \sigma T^4$. However, an amount $(1 - \varepsilon_2)\varepsilon_1\ A_1\ \sigma T^4$ will be re-emitted by surface 2 in the direction of surface 1, which will then absorb an additional amount $\varepsilon_1(1 - \varepsilon_2)$ $\varepsilon_1\ A_1\ \sigma T^4$ and re-emit an amount $(1 - \varepsilon_1)(1 - \varepsilon_2)\ \varepsilon_1\ A_1\ \sigma T^4$. Of this, an amount $\varepsilon_2(1 - \varepsilon_1)(1 -$

$\varepsilon_2)\ \varepsilon_1\ A_1\ \sigma T^4$ will be absorbed by surface 2 and an amount $(1 - \varepsilon_2)(1 - \varepsilon_1)(1 - \varepsilon_2)\ \varepsilon_1\ A_1\ \sigma T^4$ will be re-emitted in the direction of surface 1, etc, etc.

We thus see that the total amount of radiation absorbed by surface 2 is the infinite geometric series:

$$A_1\ \sigma T^4\ \{\varepsilon_1\varepsilon_2\ [1 + (1 - \varepsilon_1)(1 - \varepsilon_2) + (1 - \varepsilon_1)^2\ (1 - \varepsilon_1)^2\ + \ldots]\} \qquad \textbf{(4.27)}$$

Hence the radiative heat transfer, per unit area, between two parallel infinite planes is:

$$q_{1\text{->}2} = \varepsilon_{eff}\ \sigma\ (T_2^4 - T_1^4) \qquad \textbf{(4.28)}$$

where

$$\varepsilon_{eff} = \varepsilon_1\varepsilon_2\ /\ (\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2) \qquad \textbf{(4.29)}$$

Example 6: The emissivity of glass is typically $\varepsilon_2 = 0.88$. Thus if the absorber plate of a solar collector has an emissivity of $\varepsilon_1 = 0.95$, eq. (**4.29**) shows that $\varepsilon_{eff} = 0.84$.

REFERENCES

[**1**] **Landau, L.D.,** and **E.M. Lifshitz**.: Statistical Physics 3rd edition part 1, (Pergamon, Oxford etc, 1980).

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[**3**] **Duffie, J.A.**, and **W.A. Beckman**: Solar Engineering of Thermal Processes, (Wiley, New York etc, 1980).

[**4**] **Mills, D.R.**: *Limits of solar selective surface performance* Applied Optics **24** (1985) 3374.

APPENDIX

Selected values of the radiation integrals $F_{0 \to \lambda}(\lambda T)$. Interpolation between these values will enable integrals to be performed to sufficient accuracy for the purposes of this course.

λT [μm K x 10^3]	$F_{0 \to \lambda}(\lambda T)$ [dimensionless]	λT [μmKx10^3]	$F_{0 \to \lambda}(\lambda T)$ [dimensionless]
2	.067	10	.914
3	.273	12	.945
4	.481	14	.963
5	.634	16	.974
6	.738	18	.981
8	.856	20	.986

HOMEWORK PROBLEMS:

1. A metal plate having the properties: $\alpha_\lambda(\lambda) = 0.95$ for wavelengths up to 2.5 μm and $\alpha_\lambda(\lambda) = 0.05$ for wavelengths above 2.5 μm is placed in earth orbit, above the atmosphere, so that its plane is normal to the direction of the incident solar radiation. Calculate its equilibrium temperature.

2. Show that the shape factor for a pair of coaxial parallel discs of radii a and b respectively, where a < b, and separated by a distance L, is given by:

$$\frac{1}{2a^2}\left[L^2 + a^2 + b^2 - \sqrt{(L^2 + a^2 + b^2)^2 - 4a^2b^2}\right]$$