Introduction to Solar Energy, Lecture 3 March 25th 2003

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RADIATION COMPONENTS AND THEIR MEASUREMENT

In the previous section we studied the geometry of what is called the *direct beam* component of the radiation, i.e. that component of the solar irradiance that reaches the collector directly from the sun's apparent position in the sky. But this does not represent all the radiation that reaches the surface. There is also a *diffuse* component that comes from other parts of the sky after multiple scattering from water vapor molecules, dust particles and clouds. On an overcast day the diffuse component dominates the direct beam, but it can be substantial even on a clear day.

The direct beam component is measured using an instrument called a *pyrheliometer*. This is essentially a small blackened disk located at the rear end of a tube that is sufficiently long to collimate only that light which originates from the vicinity of the sun. The temperature of the blackened disk, as it is warmed by the incoming radiation, is measured by a thermopile whose output in mV has been calibrated against some reference standard so as to be interpretable as a radiation flux in units W m^{-2}. Now, the solar disk subtends an angle of about 0.5^0, but for practical reasons - don't forget that the pyrheliometer must track the apparent movement of the sun across the sky - the instrument has an acceptance angle of about 5.0^0. i.e. what it interprets as "direct beam" irradiance includes a certain amount of so-called *circumsolar* radiation from the sun's halo. Depending upon how clear the atmosphere is, the sun’s disk can appear enlarged to a considerable degree. There is consequently a certain degree of arbitrariness in the precise definition of the direct beam component.

A similar blackened disk mounted inside a small quartz hemispherical dome is the basis of an instrument known as a *pyranometer*. This device, when mounted horizontally, detects light originating from all parts of the sky. This so-called *global horizontal* irradiance provides an indirect measure of the diffuse component if we subtract the vertical component of the direct beam irradiance. The two kinds of instrument are illustrated schematically in **Fig. 1**. The pyranometer and the pyrheliometer, if carefully maintained, provide the most accurate method for measuring the direct beam and diffuse components of the sun's radiation. Other kinds of instrument exist, using photovoltaic cells and/or shading rings, but for a variety of reasons they are not very accurate for purposes of long-term *insolation* (i.e. solar irradiance) monitoring.

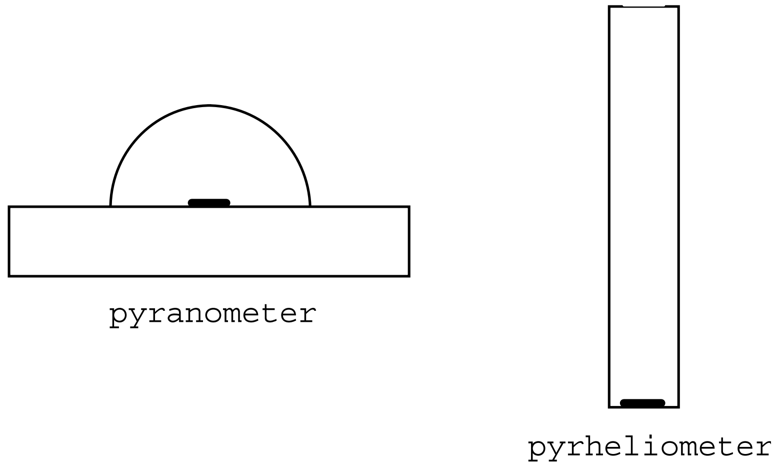

Figure 1: Schematic illustrations of thermopile instruments used for measuring the global horizontal and direct beam components of solar irradiance

If I_b and I_h denote, respectively the direct beam and the global horizontal components of the insolation, the diffuse component I_d is obtained from the difference:

$$I_d = I_h - I_b \cos \theta_z \qquad \textbf{(3.1)}$$

where θ_z denotes the *zenith angle* of the sun, i.e. the angle between the beam component and the vertical.

IRRADIANCE ON AN INCLINED PLANE

The total irradiance on an arbitrarily orientated inclined plane will be the sum of three components: A direct beam component, a sky diffuse component (from that portion of the sky seen by the plane) and a ground diffuse component. The latter, being radiation re-reflected from the ground, becomes increasingly large the more tilted the plane becomes relative to the horizontal, and can assume a critical importance in the contribution of windows to the energy balance in a building.

As first approximations, both sky and ground diffuse radiation may be treated as being *isotropic*. This is obviously not a good approximation if there are scattered clouds or if the plane in question receives radiation reflected from a lake, etc. But it is a useful approximation for making quick estimates about the performance of solar energy-collecting systems. For more reliable long-term predictions various empirical models have been developed that take into account the anisotropy of the diffuse component. **Ref.** [**1**] is one such model that is popular among many solar energy researchers.

One usually characterizes the ground reflectance by an effective *albedo* parameter ρ. The ground is then regarded as being an isotropic hemispherical source of intensity $\rho\ I_h$. The total irradiance on a plane tilted at an angle β to the horizontal is:

$$I_{tot} = I_b \cos\theta_n + I_d (1 + \cos\beta)/2 + \rho\, I_h (1 - \cos\beta)/2 \qquad \textbf{(3.2)}$$

where θ_n is the angle the solar beam makes with the normal to the inclined plane, and the sky diffuse component I_d is given by eq. (**3.1**). One notices that as the tilt angle β goes to zero, the ground reflected component drops out, and eq. (**3.2**) reduces to eq. (**3.1**).

Eq. (**3.2**) can be used together with the algebraic expressions in **Table 1** of the previous lecture in order to obtain approximate instantaneous values of the total irradiance on several kinds of stationary and moving surfaces, provided I_b and I_h are known and that some estimate can be made regarding the appropriate value to take for the ground albedo ρ. In practice, hourly average values of I_h and I_b are available for many sites around the world. For each hour of the year one may compute the appropriate values of the various angles we have discussed and then use measured values of direct beam and horizontal global insolation to compute the corresponding total irradiance on any plane of interest.

Table 1, for example, shows some derived daily average values for the tracking and fixed planar collectors discussed above, as derived from data measured in Beersheba during 1989. A ground albedo of ρ = 0.3, appropriate to desert surroundings, has been assumed, [for grasslands a typical albedo might be ρ = 0.25 and for freshly fallen snow the albedo can be as high as about ρ = 0.9]. An isotropic model has been used for the diffuse component.

	Fixed Surfaces			Tracking Surfaces			
	Horizontal	Southward tilt = latitude	Southward vertical	NS axis horizontal	EW axis horizontal	Polar axis	Dual-axis
MAR	5.10	5.74	3.90	6.43	5.75	6.95	6.96
JUN	8.08	7.20	2.74	10.15	8.30	9.62	10.25
SEP	6.30	6.93	4.28	8.13	6.96	8.66	8.68
DEC	3.27	5.00	4.91	4.59	5.57	5.93	6.39
ANN	5.74	6.24	3.92	7.37	6.64	7.81	8.07

Table 1: Mean daily irradiance [kWh/m^2] on fixed and tracking surfaces at various times of the year for Beersheba, 1989

Although **Table 1** was intended to serve primarily as an example of the output that can be obtained by using the foregoing equations with a set of direct beam and global horizontal insolation data, there are a number of important general conclusions that may be drawn from it, as regards the efficacy of the various kinds of fixed and tracking collectors at different seasons of the year. Specifically: (a) Two-axis tracking (obviously) collects the maximum amount of available solar energy at all times. (b) Polar axis tracking collects only 3% less irradiance, on an annual basis, than does full 2-axis tracking. (c) A collector tracking about a NS horizontal axis receives more solar energy on an annual basis, but less in winter, than one which tracks about an EW horizontal axis. (d) A south-facing fixed collector with tilt angle = latitude receives more solar energy on an annual basis, but less in summer, than one with its aperture plane horizontal.

In order to see how the various contributions to the total irradiance on a given surface change throughout the year, it is instructive to consider the beam, sky diffuse and ground reflected components on a south-facing vertical surface. Once again an albedo of $\rho = 0.3$ and isotropic diffuse radiation are assumed. **Table 2** shows these figures for each season of the year.

	Direct beam	Sky diffuse	Ground reflected	Total
MAR	2.16	0.98	0.77	3.90
JUN	0.44	1.09	1.21	2.74
SEP	2.48	0.85	0.94	4.28
DEC	3.99	0.44	0.49	4.91

Table 2: Mean daily irradiance [kWh/m^2] components on a vertical, south-facing surface, at various times of the year, for Beersheba 1989

Table 2 also contains some important general implications for the design of buildings that maximize the year-round comfort conditions inside. Two important conclusions that may be drawn from **Table 2** are: (a) South-facing windows are important for solar heating in winter. (b) In summer relatively little of the total irradiance on a south-facing window is from the direct beam component. This indicates that shading devices, with geometries designed to prevent the direct beam component from entering the window will not suffice to block most of the unwanted solar input: The windows must be covered entirely. Similarly, in that the diffuse component is approximately isotropic, all windows need to be covered in summer - not just the south-facing ones.

PASSIVE SOLAR HEATING OF BUILDINGS

As an example of how a knowledge of the seasonal amounts of solar radiation incident upon variously orientated surfaces is useful, let me discuss the use of solar energy to heat living spaces in winter: so-called *passive solar heating*. The term "passive" is employed to distinguish this kind of approach from that of so-called *active solar heating*, in which "active" elements, such as solar collectors, pumps, air ducts, etc., are added to the building. In general, passive solar heating is simpler and economically superior to active solar heating.

The basic idea is to design the building so that it fulfils the following 3 criteria:

1. The so-called "building envelope" (i.e. walls and roof) should be *well insulated*, in order to minimize heat loss from the interior in winter, and to minimize the unwanted heat gain from the exterior in summer.
2. The *orientation of windows* should be chosen so as to allow solar radiation easy access in winter but little, if any, access in summer. In practice, this means large equator-facing (i.e. south-facing in the Northern Hemisphere, north-facing in the Southern Hemisphere) windows for winter solar penetration, and small, or no, east-facing or west-facing windows to minimize unwanted summer solar energy collection.
3. The inclusion of so-called *thermal mass* within the building envelope. The purpose here is to store the daily intake of solar energy in winter so that the interior of the building remains warm at nighttime. Thermal mass can take the form of any material that has a substantial heat capacity (such as concrete or bricks).

Now, we need to be able to calculate such matters as: How large should the windows be? How thick should the insulation be?; How much thermal mass do we need?

Steady State Thermal Analysis

As is well known from elementary physics courses, there are three basic physical mechanisms that allow an object to gain or lose energy: *conduction*, *convection* and *radiation*. The basic equation for conduction was discovered by Newton:

$$q'_{cond} = (\kappa/d)\,(T_2 - T_1) = U_{cond}\,\Delta T \qquad \textbf{(3.3)}$$

where q'_{cond} is the rate of heat flux from a region at temperature T_2 to a region at temperature T_1, measured in W m^{-2}, κ is the thermal conductivity of the material through which the heat is flowing, measured in W m^{-1} K^{-1}, and d is the thickness of the material, also measured in m. Building components (bricks, windows, plaster boards, etc.) may therefore be characterized by a conductive heat loss coefficient ("U-value") defined as the thermal conductivity divided by the thickness of the component.

Example: Glass has a thermal conductivity of 0.81 W m^{-1} K^{-1}. A windowpane of thickness 3 mm therefore has a U-value of 0.81 / 0.003 = 270 W m^{-2} K^{-1}. This means that a temperature difference of, say, 10 ^{o}C (e.g. 20 ^{o}C indoors and 10 ^{o}C outside) would give rise to a conductive heat flow of 2.7 kW per m^2 of glazing area. However, as we shall soon see, this is not the entire story as regards a window.

Now, it is convenient to define a *thermal resistance* (R-value) for a building component, as the *reciprocal* of the U-value. It's importance lies in the fact that it enables eq. (**3.3**) to be converted into one that is mathematically equivalent to Ohm's law for electric circuit components:

$$\Delta T = (1 / U_{cond}) \, q'_{cond} = R \, q'_{cond} \qquad (\mathbf{3.4})$$

where the temperature difference is the analog of the electrical potential difference ΔV and the heat flux is equivalent to the electric current I. Thanks to eq. (**3.4**), for a compound building component (e.g. a multi-layer wall with windows) the R-values of the various parts may be combined like electrical resistances (in series or parallel, as the case may be).

Example: A 10 cm thickness of brick wall ($k = 0.72$ W $m^{-1}K^{-1}$) is covered on each face with a 1 cm thick layer of plaster ($k = 0.81$ W m^{-1} K^{-1}). The total R-value is: $(0.81/0.01)^{-1} + (0.72/0.10)^{-1} + (0.81/0.01)^{-1} = 0.164$ [W m^2 $K^{-1}]^{-1}$. Hence, for this wall section $U = 1/R = 6.11$ W m^2 K^{-1}. Tables of conductive U-values for a wide range of building materials are given in Ref. [**2**].

Convection and Radiation

Provided temperatures are not too high – which is generally the case for the heating and cooling of buildings (as will be discussed in more depth in a later lecture) - both convective and radiative heat flux can also be approximated by equations similar to eq. (**3.3**). We may therefore also characterize building components according to their radiative U-values (related to their color: dark colors tending to absorb radiation better than light ones) and their convective U-values (related to their thermal behavior when exposed to wind).

The convective heat losses are allowed for in a semi-empirical manner. The boundary layer of air on an interior surface contributes a significant contribution to the effective R-value of that component. This is particularly true for the stationary layer trapped up against a ceiling, but it also holds true for a slowly-moving air layer in contact with a wall surface. Such relatively stationary air films add an approximate contribution of $R = 0.13$ [W m^{-2} $K^{-1}]^{-1}$.

Air gaps within building components, such as the space contained in a cavity wall or between the glass panes in double glazed window, also contribute to the effective R-value of the component. A typical value of the contribution of such an air gap is approximately R = 0.16 [W m^{-2} K^{-1}]$^{-1}$.

Finally, even the thin layer of air up against a surface exposed to high wind, also contributes, not insignificantly, to its effective R-value. Empirical expressions and tables for these convective corrections to the R-values of building components are given in Ref. [**2**]. For example, for a wind speed of 16 km h^{-1}, the exterior air film contributes R = 0.04 [W m^{-2} K^{-1}]$^{-1}$, approximately. Thus if we add this contribution, together with the R = 0.16 [W m^{-2} K^{-1}]$^{-1}$ contribution from the interior air film, to the R = 0.0037 [W m^{-2} K^{-1}]$^{-1}$ we calculated for a 3 mm thick glass window, we arrive at a total effective thermal resistance of R = 0.2037 [W m^{-2} K^{-1}]$^{-1}$, or U = 4.9 W m^{-2} K]$^{-1}$. This represents a considerably lower rate of heat loss from a window than the purely conductive contribution we calculated previously.

Finally, it is important to realize that the true temperature difference between the inner and outer surfaces of a building component is not necessarily the difference of the interior and exterior air temperatures. This is because the absorptance of the surface to visible radiation and its emittance in the infrared will both effect the energy balance. That dark exterior surfaces heat up when exposed to sunshine is a most common manifestation of this effect. The actual temperature that a given exposed surface achieves naturally also depends on how rapidly it is cooled by wind. Ref. [**2**] tabulates the appropriate thermal properties that should be used when computing accurate cooling loads for buildings, in particular the loads associated with surfaces of various finish and color.

Air Infiltration

In addition to heat loss through the *material* of the building, one must also consider the energy content of warm air that passes through openings in the building envelope. Such openings might be doors or windows that are not tightly sealed, or not infrequently, imperfections in the construction itself. Air infiltration is often measured (or assessed) in units of air *changes per hour* (ACH). Taking air to have a density of 1.2 kg m^{-3} at standard atmospheric pressure, and a heat capacity of 1000 J kg]$^{-1}$ K]$^{-1}$, it is easy to see that an infiltration rate of n ACH will give rise to an energy loss at the rate of:

$$nV/3 \quad W\ K^{-1} \qquad (\mathbf{3.5})$$

where V is the interior air volume of the building.

A useful exercise

In order to appreciate the relative size of these various effects; consider the schematic house shown in **Fig. 2**.

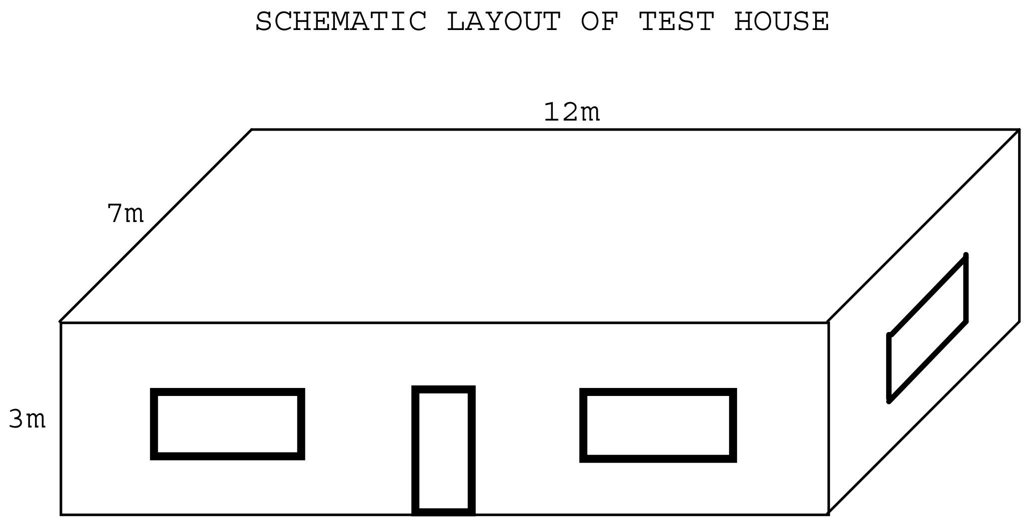

Figure 2: Schematic house for computing heat losses

The walls are of so-called *cavity construction* (two concrete slabs each of thickness 8 cm separated by an air gap of width 4 cm). The roof consists of a horizontal concrete slab of thickness 10 cm. There are 6 windows of area 1.5 m^2 each (only 3 windows are visible in the figure), and one wooden door of area 2 m^2. We are interested in the relative effectiveness of adding insulation to the various components of this house in order to reduce winter heating costs. e.g. sealing air leakage gaps under doors and windows, adding polystyrene insulation to the roof and walls, using double rather than single glazed windows.

Consider the heat loss from the <u>entire</u> building envelope during a 24 hour winter day in which the mean exterior temperature is 10^{o}C and the interior is maintained (using heating) at 18^{o}C. Compute the contributions to the total lost heat load from: (a) 3 ACH [a typical "leaky" house], (b) 1 ACH [a well "sealed" house], (c) no wall insulation, (d) 3 cm of wall insulation [Typical of the Israeli building code], (e) 6 cm of wall insulation, (f) no roof insulation, (g) 5 cm of roof insulation [typical of the Israeli building code], (h) 10 cm of roof insulation, (i) single glazed windows, (j) double glazed windows.

Ignore the exterior color of walls and roof and conductive heat loss through the doors (which we shall assume to be well insulated). For each component compute the overall U-value (including interior and exterior air films) and multiply by the total area of material involved and by the temperature difference. Use the following values for the thermal conductivity of materials: concrete k = 2.1 W m^{-1} K^{-1}, polystyrene k = 0.04 W m^{-1} K^{-1}, glass k = 0.81 W m^{-1} K^{-1}, and the following convective conductances for air; exterior air film on wall or roof U = 25 W m^{-2} K^{-1}, interior air film on wall or ceiling U = 7.7 W m^{-2} K^{-1}, enclosed air gap in cavity wall or double glazing U = 6.3 W m^{-2} K^{-1}.

Consider the result of various combinations on the heat loss of the entire house. The economic value of employing any of these various strategies to reduce winter heating bills could be calculated from a knowledge of the costs of the various materials.

Dynamic Properties of Building Components

Thus far we have discussed the steady-state thermal properties of a house. However, since the house will experience different exterior temperatures between day and night, and when the weather changes suddenly, it is important to understand its response under such circumstances. As an extreme example, a hypothetical house made entirely from polystyrene insulation and glass windows would rapidly become unbearably hot soon after sunrise, and would cool off quite quickly after sunset. On the other hand, if there were enough interior mass to absorb the incoming solar energy the temperature rise and subsequent fall would be slower and less extreme. The purpose of the present section is to enable us to understand the balance that must be struck between insulation and thermal mass so as to produce acceptable comfort conditions.

We start with the basic equation describing the temporal and spatial passage of heat (assumed 1-dimensional for simplicity) through a building component:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

(3.6)

where T = T(x,t) is the temperature at time t and position x (within the slab), and α is the *thermal diffusivity*, defined as:

$$\alpha = \kappa / \rho C \qquad \textbf{(3.7)}$$

where κ is the thermal conductivity, ρ is the density and C is the heat capacity of the material. Now α has the dimensions [m^2 t^{-1}] and we can, accordingly, define a characteristic time τ_1 based on the thickness of the wall (let us say) d:

$\tau_1 = d^2/\alpha$ **(3.8)**

For a concrete wall of thickness 20cm (taking $\rho = 2400$ kg m^{-3}, $C = 864$ J kg^{-1} K^{-1} and $\kappa = 2.1$ W m^{-1} K^{-1}), the characteristic time τ_1 is seen to be about 11 hours. The physical interpretation of τ_1 is that it represents the response of the wall to a *sudden change* after a lengthy period of stable weather conditions, i.e. when the left-hand side of the differential equation (**3.6**) suddenly changes from being close to zero.

A more interesting question concerns the *diurnal* response of the wall during stable weather conditions. For example, many Israeli homes are notoriously hot *at night* in summer time! What is happening there is that a wave of daytime solar heating gradually penetrates the building envelope and (over)heats the residents after the sun has set. Let us examine the physics of this kind of heat wave in order to see what should be done so that buildings will not exhibit problems of this kind. We can approximate the heat wave as:

$$T(x,t) = T_o \exp [\, i\omega\, t - x/\lambda] \quad \textbf{(3.9)}$$

where T_o is the amplitude of the temperature at the wall's outer surface ($x = 0$), and $\omega = 2\pi/(24 \text{ hrs} \times 60 \text{ min} \times 60 \text{ sec})$ is the angular frequency of the diurnal heat wave. Insertion of this solution into eq. (**3.6**) reveals the parameter λ to be a complex number, with equal real and imaginary parts:

$$\mathcal{R}\lambda = I\lambda = \sqrt{\frac{2\alpha}{\omega}}$$

(3.10)

Hence the general solution, as given by eq. (**3.9**), becomes:

$$T(x,t) = T_o \exp\left(-\sqrt{\frac{\omega}{2\alpha}}\, x\right) \exp i\omega\left(t - \sqrt{\frac{x^2}{2\alpha\omega}}\right)$$

(3.11)

We thus see that the *time lag* τ_2 associated with the passage of the heat wave through the entire wall of thickness d is:

$$\tau_2 = \sqrt{\frac{d^2}{2\alpha\omega}}$$

(3.12)

which, for a concrete slab of thickness d = 20cm, takes the value τ_2 = 4.6 hours.

In addition to this time lag factor, which can be tailored by appropriate choice of building materials (such as thermal insulation), eq. (**3.11**) also reveals an important attenuation factor:

$$\frac{T(x=d)}{T(x=0)} = \exp\left(-\sqrt{\frac{\omega}{2\alpha}}\, d\right) \quad \textbf{(3.13)}$$

which amounts to a factor of about 0.3 for a 20 cm thick concrete wall.

We can thus see the quantitative manner in which thermal mass in the building envelope both attenuates and delays the incoming diurnal heat wave caused by the cyclic daily heating of the building exterior by summer sunshine. For further discussion of how actual passive solar calculations may be performed on a seasonal basis, Ref [**3**] provides an excellent physics-based discussion.

REFERENCES

[**1**] **R. Perez, P. Ineichen, R. Seals, J. Michalsky** and **R. Stewart**, Solar Energy **44** (1990) 271.

[**2**] **ASHRAE** *Handbook of Fundamentals*, (American Society of Heating, Refrigeration and Air Conditioning Engineers, Atlanta GA, 1977) and later editions.

[**3**] **J.M. Gordon** and **Y. Zarmi**, "Analytic model for passively-heated solar houses - I. Theory", Solar Energy **27** (1981) 331-342; and "ditto - II. Users Guide", ibid pp 343-347.

HOMEWORK PROBLEMS

1. The data file provided for this exercise contains average meteorological data for every hour of an entire “typical” year at Sede Boqer. Adapt the computer program you wrote after lecture 2, in order to access this data and calculate the total solar irradiance on a plane surface with any given orientation. Input parameters should be the tilt angle and azimuthal angle of the plane surface and the average albedo of the surrounding ground.

2. Use your program to calculate the monthly average total radiation, for each month of the year, for a south-facing surface with a tilt angle of 60^0 to the horizontal, assuming a value of 0.3 for the ground albedo. CAUTION: You must include flags that set the direct beam

component of the solar radiation equal to zero if the sun is *behind* the plane of the collecting surface!

3. Consider a composite wall that is fabricated from two slabs of different materials. One part has thickness t_1 and thermal diffusivity α_1. The other part has thickness t_2 and thermal diffusivity α_2. (a) Calculate the time lag and attenuation that are associated with the passage of a sinusoidal heat wave through the wall. (b) Calculate the numerical values if one part of the wall is a concrete slab of thickness 20 cm and the other part is a sheet of polystyrene of 5 cm thickness. (c) How does the direction in which the heat wave travels affect your result?