Composite Edge States in the $\nu = \frac{2}{3}$ Fractional Quantum Hall Regime

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A generalized $\nu = \frac{2}{3}$ state, which unifies the sharp edge picture of MacDonald with the soft edge picture of Chang and Beenakker is presented and studied in detail. Using an exact relation between correlation functions of this state and those of the Laughlin $\nu = \frac{1}{3}$ wave function, the correlation functions of the $\nu = \frac{2}{3}$ state are determined via a classical Monte Carlo calculation, for systems up to fifty electrons. It is found that as a function of the slope of the confining potential there is a sharp transition of the ground state from one description to the other. The experimental implications are discussed.

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The prolific research on the fractional quantum Hall effect in the last decade has led to a very good understanding of the bulk properties of the fractional quantum Hall liquid [1]. The nature of the edge states in this regime, however, is far from being well understood [2]. Particularly intriguing is the situation for $\nu = \frac{2}{3}$, where several theories have been proposed to describe the edge states. One picture, due to MacDonald [3], is based upon a wave function proposed by Girvin [4], which, due to particle-hole symmetry, consists of a droplet of holes in the $\nu = \frac{1}{3}$ Laughlin state [5] superimposed on a droplet of electrons in the $\nu = 1$ state. This wave function has indeed been shown to be an excellent description of the exact ground state for a system with a small number of electrons [6–8]. For example, Greiter [8] quotes an overlap of 0.9990 between the exact ground state and the Girvin wave function for a system of eight electrons in spherical geometry. This $\nu = \frac{2}{3}$ wave function supports two different edges [3], one at the edge of the hole droplet (of charge $q = -e/3$), and the other at the edge of the $\nu = 1$ electron droplet (of charge $q = e$). On the other hand, a very different edge structure was suggested by Chang and by Beenakker [9], and elaborated on by Chklovskii, Shklovskii, and Glazman [10] in a more general context; they argued that for a smooth enough potential an incompressible $\nu = \frac{1}{3}$ state will nucleate near the edges of the system, leading again to two edge branches, but this time of charge $q = e/3$ each [11].

Recently it was argued [7] that tunneling into a $q = \pm e/3$ edge state will reduce the tunneling amplitude by a factor of $1/N$ relative to the integer case. Hence, tunneling measurements through a small system in the fractional quantum Hall regime offer the exciting possibility of directly probing the composition of the edge structure of the system. At zero or low magnetic fields the conductance consists of a series of well separated peaks [12], each corresponding to an electron tunneling through a specific state. In the first scenario, where a single edge state carries a fractional charge, one would expect that half of the peaks will be suppressed, giving a clear signature of the composition of the edge states. In the second scenario, where both edges carry a fractional charge, all the peaks would be suppressed, resulting in a very low conductance signal. Until now there has been no quantitative theoretical understanding of the experimental circumstances required to explore each scenario.

In this work we study quantitatively the nature of the ground state and the corresponding edge states in the $\nu = \frac{2}{3}$ regime. Generalizing the Girvin wave function to incorporate the possibility of a $\nu = \frac{1}{3}$ strip near the edge of the sample, the correlation functions in this generalized state are expressed exactly in terms of correlation functions calculated in the $\nu = \frac{1}{3}$ Laughlin wave function. Using the mapping into a classical one-component two-dimensional plasma [5] we calculate those correlation functions using the classical Monte Carlo method [13] for up to fifty electrons. The resulting $\nu = \frac{2}{3}$ correlation functions enable us to calculate the energy of the state for arbitrary electron-electron interactions and confining potential. We find that as a function of the slope of the confining potential, the ground state makes a sharp transition from the Girvin form to the Chang-Beenakker form. This calculation suggests that for heterostructures where the gates are not too far from the two-dimensional electron gas, the suppression of half of the peaks, in the first scenario above, should be observable. In addition, information about the actual distance between the two edges, which is a crucial ingredient of recent edge state theories [2], is obtained.

The ground state of N electrons in a radially symmetric system in the $\nu = \frac{1}{3}$ fractional quantum Hall regime can be approximated very well by the Laughlin wave function [5],

$$\Psi^{(1/3)}(z_1, \ldots, z_N) = \prod_{i<j} (z_i - z_j)^3 e^{-\sum_i |z_i|^2/4}$$
$$\triangleq \sum_{\{i_1, \ldots, i_N\}} C^{(N)}_{\{i_1, \ldots, i_N\}} \mathbf{a}^\dagger_{i_1} \cdots \mathbf{a}^\dagger_{i_N} |0\rangle , \quad (1)$$

where z_i denotes the complex coordinates of the ith particle in the plane, and all lengths are expressed in units of the magnetic length, $l_H \equiv \sqrt{\hbar c/eH}$. $\triangleq$ denotes a second-quantization representation, where $\mathbf{a}^\dagger_n$ creates an electron in a first Landau-level state of angular momentum n, described by the single-particle wave function $\phi_n(z) = z^n \times \exp(-|z|^2/4)/\sqrt{2\pi 2^n n!}$. The sum is over all permutations of N distinct integers which sum up to the total an-

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gular momentum $3N(N-1)/2$, and the $C^{(N)}_{\{i_1,\ldots,i_N\}}$ can, in principle, be obtained by expanding the first product. In the second-quantization representation, the particle-hole symmetric wave function, introduced by Girvin to describe the $\nu=\frac{2}{3}$ state [4], is

$$\Psi^{(2/3;N_h)}(z_1,\ldots,z_N) \triangleq \sum_{\{i_1,\ldots,i_{N_h}\}} C^{(N_h)}_{\{i_1,\ldots,i_{N_h}\}} a_{i_1}\cdots a_{i_{N_h}} a^\dagger_1 \cdots a^\dagger_{N+N_h}|0\rangle . \tag{2}$$

The yet undetermined number of holes N_h must be chosen to minimize the energy.

In order to allow for the possibility of a $\nu=\frac{1}{3}$ state nucleating along the edge of the sample, we start with the Laughlin wave function with an inside hole of size L [5],

$$\Psi^{(L;1/3)}(z_1,\ldots,z_N) = -\prod_i z_i^L \prod_{i<j}(z_i-z_j)^3 e^{-\sum_i |z_i|^2/4} \triangleq \sum_{\{i_1,\ldots,i_N\}} D^{(N;L)}_{\{i_1,\ldots,i_N\}} a^\dagger_{i_1+L}\cdots a^\dagger_{i_N+L}|0\rangle , \tag{3}$$

where the sum is over the same sets as in (1), and $D^{(N;L)}$ can again, in principle, be evaluated by expanding the products. With the above wave function describing electrons with $\nu=\frac{1}{3}$ correlations along the edge of the sample, we write a generalized $\nu=\frac{2}{3}$ state,

$$\Psi^{(2/3;N_h,L,N_2)}(z_1,\ldots,z_N) \triangleq \sum_{\{i_1,\ldots,i_{N_h}\}} \sum_{\{j_1,\ldots,j_{N_2}\}} C^{(N_h)}_{\{i_1,\ldots,i_{N_h}\}} D^{(N_2;L)}_{\{j_1,\ldots,j_{N_2}\}} a_{i_1}\cdots a_{i_{N_h}} a^\dagger_1\cdots a^\dagger_{N_1+N_h} a^\dagger_{j_1+L}\cdots a^\dagger_{j_{N_2}+L}|0\rangle . \tag{4}$$

This wave function is schematically depicted in Fig. 1. It depends on three integer parameters. Out of the N electrons, N_1 are described by the Girvin wave function (2), with N_h holes. The remaining $N_2=N-N_1$ electrons nucleate into a $\nu=\frac{1}{3}$ strip along the edge of the sample, with minimal angular momentum L ($L>N_1+N_h$).

Our task now is to find the set of parameters that minimizes the energy for a given confining potential and interactions. To this end we need to calculate the one-particle and two-particle correlation functions for this state. In principle, of course, if one can obtain the coefficients $C^{(N_h)}_{\{i_1,\ldots,i_{N_h}\}}$ and $D^{(N_2;L)}_{\{j_1,\ldots,j_{N_2}\}}$, all correlation functions for the $\nu=\frac{2}{3}$ state [Eq. (4)] should be easily evaluated. This, however, can only be achieved for a system of a very small number ($N\le 6$) of particles [7,14]. Correlation functions for the $\nu=\frac{1}{3}$ Laughlin-like wave function (3), on the other hand, can be easily calculated for a large number of particles, using a mapping into a classical statistical problem [5]. Such a mapping, however, is not possible for the Girvin wave function (2), or the generalized wave function (4).

The most important step in this work is expressing the correlation functions for the generalized $\nu=\frac{2}{3}$ wave function in terms of correlation functions for the Laughlin-like wave functions for $\nu=\frac{1}{3}$. Using the explicit form of this wave function (4), we find

$$\begin{aligned}
\rho_1^{(2/3;N;N_h,L,N_2)}(r) &= \rho_1^{(1;N_1)}(r) - \rho_1^{(1/3;N_h)}(r) + \rho_1^{(1/3;N_2;L)}(r) ,\\
\rho_2^{(2/3;N;N_h,L,N_2)}(r_1,r_2) &= \rho_2^{(1;N_1)}(r_1,r_2) + \rho_2^{(1/3;N_h)}(r_1,r_2) + \rho_2^{(1/3;N_2;L)}(r_1,r_2)\\
&\quad - \rho_1^{(1;N_1)}(r_1)\rho_1^{(1/3;N_h)}(r_2) - \rho_1^{(1;N_1)}(r_2)\rho_1^{(1/3;N_h)}(r_1) + \rho_1^{(1;N_1)}(r_1)\rho_1^{(1/3;N_2;L)}(r_2)\\
&\quad + \rho_1^{(1;N_1)}(r_2)\rho_1^{(1/3;N_2;L)}(r_1) - \rho_1^{(1/3;N_h)}(r_1)\rho_1^{(1/3;N_2;L)}(r_2) - \rho_1^{(1/3;N_h)}(r_2)\rho_1^{(1/3;N_2;L)}(r_1)\\
&\quad + 2\,\mathrm{Re}\left[\sum_{i=0}^{3(N_h-1)}\sum_{j=0}^{N_1-1} \langle n_i\rangle_{1/3}^{(N_h)} \phi_i^*(r_1)\phi_i(r_2)\phi_j^*(r_2)\phi_j(r_1)\right]\\
&\quad - 2\,\mathrm{Re}\left[\sum_{i=L}^{L+3(N_2-1)}\sum_{j=0}^{N_1-1} \langle n_i\rangle_{1/3}^{(N_2;L)} \phi_i^*(r_1)\phi_i(r_2)\phi_j^*(r_2)\phi_j(r_1)\right]\\
&\quad + 2\,\mathrm{Re}\left[\sum_{i=L}^{L+3(N_2-1)}\sum_{j=0}^{3(N_h-1)} \langle n_i\rangle_{1/3}^{(N_2;L)} \langle n_j\rangle_{1/3}^{(N_h)} \phi_i^*(r_1)\phi_i(r_2)\phi_j^*(r_2)\phi_j(r_1)\right] .
\end{aligned} \tag{5}$$

The single-particle distribution function ρ_1, normalized such that its integral is N, is simply expressed as the sum of the three distribution functions of the N_1 electrons in the $\nu=1$ state ($\rho_1^{(1;N-N_2)}$), that of the N_2 electrons in the strip of $\nu=\frac{1}{3}$ state ($\rho_1^{(1/3;N_2;L)}$), and (minus) that of the N_h holes in the $\nu=\frac{1}{3}$ state ($\rho_1^{(1/3;N_h)}$). The two-particle correlation function, ρ_2, here normalized to $N(N-1)$, is far more complicated. Nevertheless, the various terms contributing to the resulting interaction energy have straightforward interpretations. The first three terms describe the contribution to the interaction energy from interactions within the three different components. The next six terms describe the direct (Hartree) interactions between the three components. The last three nontrivial terms correspond to the exchange and correlation interactions between the different components.

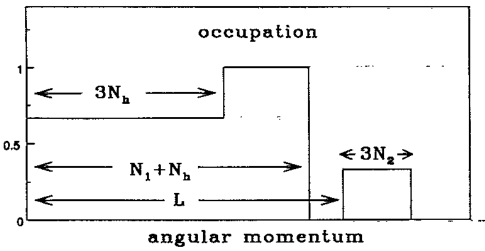

FIG. 1. Schematic representation of the generalized $\nu=\frac{2}{3}$ state (4). Out of the N electrons, N_1 are described by the Girvin wave function, consisting of N_h holes in the $\nu=\frac{1}{3}$ state in the background of N_1+N_h particles in the $\nu=1$ state (2). The remaining N_2 electrons nucleate into a $\nu=\frac{1}{3}$ strip along the edge of the sample.

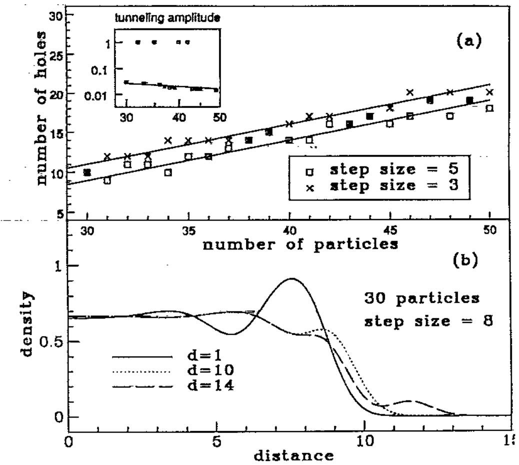

FIG. 2. (a) The number of holes that minimizes the energy of the Girvin wave function (2) for two different sizes of step potentials. The straight lines correspond to $N_h \propto N/2$, leading to an N-independent edge size. The inset shows the tunneling amplitude, as estimated from the occupation of the relevant state, as a function of N, on a log-log plot. The suppression of at least half of the peaks is evident and agrees very well with the theoretically predicted $1/N$ dependence [7]. (b) The density profile of the ground states of $N=30$ electrons for three different potential slopes. The existence of a $\nu=1$ region is evident for the highest slope, while for the other two slopes an incompressible $\nu=\frac{1}{3}$ strip is formed along the edge.

Equation (5) enables us to express the one- and two-particle correlation functions for the $\nu=\frac{2}{3}$ state in terms of quantities evaluated for the $\nu=\frac{1}{3}$ states [Eq. (1) and Eq. (3)]. We calculate the $\nu=\frac{1}{3}$ correlation functions using the classical Monte Carlo method [13,15]. Having obtained the correlation function for the generalized $\nu=\frac{2}{3}$ state (4), its energy can easily be evaluated for any choice of interactions and confining potential. In order to be as close to the experimental situation as possible, we have chosen a Coulomb interaction, $U(r)=e/\epsilon r$. The confining potential was chosen to be zero for $r<r_0-d/2$ and then to rise linearly from zero to S, over a length d. For $r>r_0-d/2$ it is constant, equal to S. The position of the midheight of the potential step r_0 is fixed so the filling factor is $\frac{2}{3}$. As discussed below, the physically relevant parameter will be the slope of the potential, S/d. d is determined experimentally by the distance of the gates from the two-dimensional electron gas, while S is determined by the amount of voltage applied to the gate, as seen by the electrons in the 2D gas. For typical Ga-As samples, the gates are 120–200 nm from the 2D gas, which corresponds to 8–12 magnetic lengths. The interaction energy $e/\epsilon l_H$ is typically 5 meV, while the boundary potential seen by the electrons is tens of meV [16]. Here we will express all energies in units of $e/\epsilon l_H$. The calculations were done for up to fifty electrons, which is a typical number in an experimental quantum dot [12].

In Fig. 2(a) we plot the number of holes N_h, which minimizes the energy for a step potential ($d=0$), for two values of $S=3$ and $S=5$. For a step potential, the ground state usually involves $N_2=0$ electrons in the $\nu=\frac{1}{3}$ strip, so it is of the Girvin type (2). The number of holes in the ground state is determined by the competition between the two contributions to the energy: The larger the number of holes, the more uniform the density, and the lower the interaction energy. On the other hand, the larger the number of holes, the larger the potential energy. Thus, as the potential becomes smoother, the number of holes may increase and a strip of electrons in the $\nu=\frac{1}{3}$ state may form near the edge.

As can be seen from Fig. 2, the number of holes in the ground state scales as $N/2$, which shows the region of density different from $\nu=\frac{2}{3}$ is independent of N, namely, an edge effect. From Fig. 2(a) one can obtain the physical distance between the edges for large enough systems, as a function of the confining potential. We find that this distance changes from $\sim 1.5 l_H$ to $\sim 2.5 l_H$ when S changes from 3 to 10. Thus, unlike the case for slowly varying confining potential [10], one cannot consider those edges as isolated from each other, and any theory should include interactions and mixing of those states.

Since the number of holes is an integer, it will change, on average, every other time an electron is added to the system. This is the source of the prediction [7] that half of the peaks for tunneling into a $\nu=\frac{2}{3}$ droplet will be suppressed. As the present calculation cannot produce the tunneling amplitudes exactly, we estimate them by their upper limit, the average occupation of the angular momentum state the electron tunnels to. In the inset of Fig. 2(a) we plot this occupation as a function of N. The suppression of more than half of the peaks is clearly observed, with the right power-law dependence on the electron number. Interestingly, the calculation suggests that sometimes the ground states of N and $N+1$ electrons differ by two holes. It remains to be seen if this is a real effect, which will result in a more dramatic reduction of the peak amplitude.

In any real system the potential will rise over a finite length scale d. We have studied the nature of the ground state as a function of d, and we found that for a given electron number N, and potential height S, there will be a transition from the ground state being of the Girvin type (2) to a state which includes electrons nucleating at the edge of the sample in the $\nu=\frac{1}{3}$ state. By varying S it is found the transition occurs at the same ratio of S/d, namely, at a given slope of the confining potential. For example, for thirty electrons the ground state evolves smoothly from $N_h=9$ and $N_2=0$, for $d=0$, to $N_h=5$ and $N_2=0$, for $S/d \simeq 1$, and then it changes abruptly to $N_h=15$, $N_2=2$, and $L=N_1+N_h+1$ [Fig. 2(b)]. Thus the two edges of the electron droplet and the hole droplet suddenly merge, and a $\nu=\frac{1}{3}$ strip forms, signaling a transition from the Girvin-MacDonald picture to the picture presented by Chang and by Beenakker [9] and Chklovskii, Shklovskii, and Glazman [10]. This strip moves away from the edge of the electron droplet ($L=N_1+N_h+1$) as S/d decreases. For example, for $S/d=0.6$ the ground state corresponds to $N_h=15$, $N_2=2$, and $L-(N_1+N_h)=20$ [Fig. 2(b)]. For forty electrons one can actually see two transitions. For $S/d \simeq 1$ the ground state changes from $N_h=12$ and $N_2=0$ to $N_h=19$ and $N_2=0$, namely, it is still described by Eq. (2), but the two edges have merged, while for $S/d \simeq 1.4$ nucleation first occurs with $N_h=18$ and $N_2=5$. This intermediate regime where the two edges merged may suggest a possible description in terms of a single $\nu=\frac{2}{3}$ edge [17].

Similar transitions have been observed for other forms of confining potentials and electron-electron interactions. As the states between which the transitions occur have identical bulk structure and are only different near the boundary of the system, the difference in their potential energy only depends on the shape of the confining potential in that region, where the confining potential can be considered linear to a good approximation [16]. As the slope of the potential in experimental systems [12] is of the order of $(1.2\text{-}3)e^2/\epsilon l_H^2$ [16], we predict that the suppression of half of the tunneling peaks should be observable in quantum dots in present high mobility structures. The closer the gates to the 2D gas, the better the chances of seeing that effect. In addition, it is predicted that as a function of the voltage applied to the gates (which changes the slope of the effective potential) the tunneling peak structure will change abruptly as this transition occurs. For high voltages half of the peaks appearing in the $\nu=1$ regime will be suppressed in the $\nu=\frac{2}{3}$ regime, while for lower voltages, as extensive tunneling into the $\nu=\frac{1}{3}$ state will occur, most or all of the peaks will be suppressed.

In conclusion, using an exact expression for the generalized $\nu=\frac{2}{3}$ state correlation functions in terms of the $\nu=\frac{1}{3}$ ones, we have been able to study quantitatively systems of relatively large numbers of electrons ($N \leq 50$). Consequently, we predict a transition in the nature of the ground state of the system as a function of the slope of the confining potential and discuss its experimental manifestation. It is hoped that this work will stimulate more experiments in this direction.

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