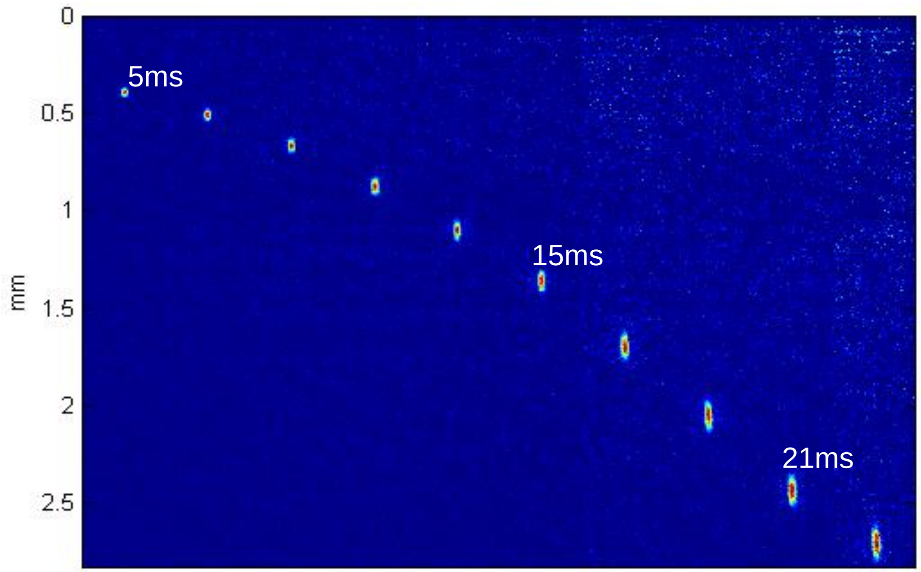
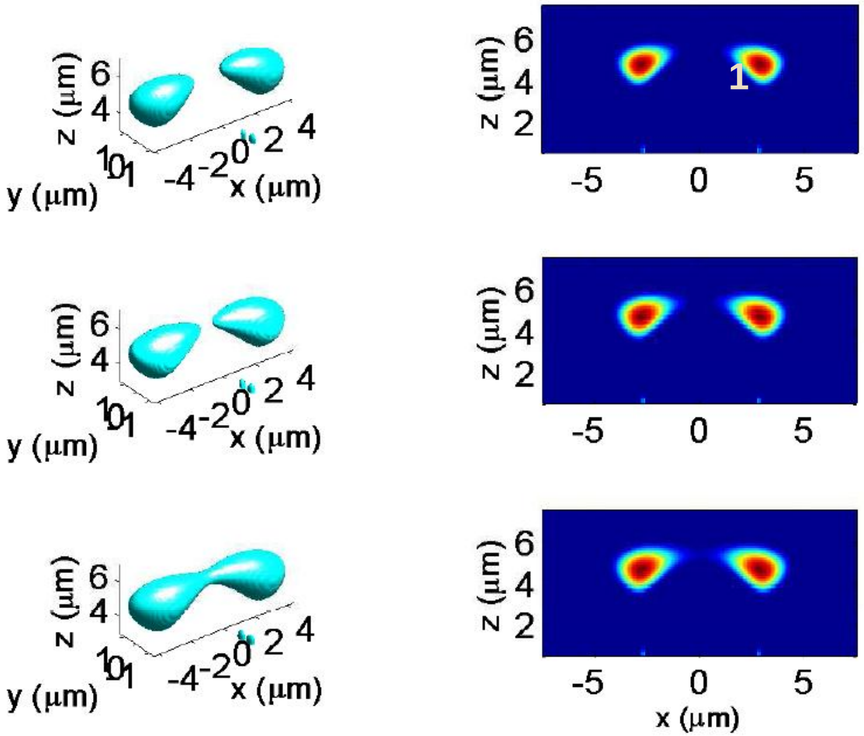
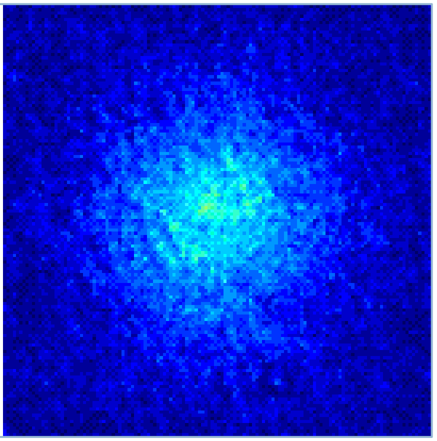






BOSE-EINSTEIN CONDENSATION IN A DOUBLE WELL

Shimon Machluf, 30/12/2010

Outline

- Bose-Einstein condensation
- Single mode solution
- Double mode simple solution
- Multi mode solution

Non-interacting Bosons

- Solution in a harmonic potential

$$\phi(\vec{r}_1, ..., \vec{r}_N) = \prod_i \varphi_0(\vec{r}_i), \qquad \text{with}$$

$$\varphi_0(\vec{r}) = \left(\frac{m\omega_{ho}}{\pi\hbar}\right)^{3/4} \exp\left(-\frac{m}{2\hbar}(\omega_x x^2 + \omega_y y^2 + \omega_z z^2)\right).$$

- The atom number

$$N - N_0 = \int_0^\infty \frac{dn_x dn_y dn_z}{\exp\left(\beta\hbar(\omega_x n_x + \omega_y n_y + \omega_z n_z)\right) - 1}.$$

$$N - N_0 = \zeta(3)\left(\frac{k_B T}{\hbar\omega_{ho}}\right)^3$$

- And the phase transition

$$T_c = \frac{\hbar\omega_{ho}}{k_B}\left(\frac{N}{\zeta(3)}\right)^{1/3} = 0.94\frac{\hbar\omega_{ho}}{k_B}N^{1/3}$$

Interacting Bosons

- The Hamiltonian

$$\hat{H} = \int d\vec{r}\, \hat{\Psi}^\dagger(\vec{r}) \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(\vec{r}) \right) \hat{\Psi}(\vec{r}) + $$
$$+ \frac{1}{2}\int d\vec{r}\, d\vec{r'}\; \hat{\Psi}^\dagger(\vec{r})\hat{\Psi}^\dagger(\vec{r'})V_{int}(\vec{r}-\vec{r'})\hat{\Psi}(\vec{r})\hat{\Psi}(\vec{r'}).$$

- Time dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\hat{\Psi}(\vec{r},t) = \left[\hat{\Psi},\hat{H}\right]$$
$$= \left[-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\vec{r}) + \int d\vec{r'}\hat{\Psi}^\dagger(\vec{r'},t)V_{int}(\vec{r'}-\vec{r})\hat{\Psi}(\vec{r'},t)\right]\hat{\Psi}(\vec{r},t)$$
$$\left\{V_{int} = g\delta(\vec{r'}-\vec{r})\right\} = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\vec{r}) + g\hat{\Psi}^\dagger(\vec{r},t)\hat{\Psi}(\vec{r},t)\right]\hat{\Psi}(\vec{r},t)$$

Single mode solution

- Assuming BEC: all the atoms in the same mode

$$i\hbar\frac{\partial}{\partial t}\Phi(\vec{r},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\vec{r}) + g|\Phi(\vec{r},t)|^2\right]\Phi(\vec{r},t).$$

- Time independent solution

$$\Phi(\vec{r},t) = \phi(\vec{r})e^{-i\mu t/\hbar}$$

$$\left[-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\vec{r}) + g\phi^2(\vec{r})\right]\phi(\vec{r}) = \mu\phi(\vec{r}).$$

Single mode numerical solution

- Solving the following differential equation

$$\Phi(t+dt) = \Phi(t) - iH\Phi(t)dt/\hbar$$

- Can be solved with RK and gives time-dependent solutions
- Ground state can be found with imaginary time

$$\Phi(t+dt) = \Phi(t) - H\Phi(t)dt/\hbar$$

  - Oscillating solutions $\Phi(t) = \sum A_j \phi_j e^{-iE_j t/\hbar}$
  - Decaying solutions $\Phi(t) = \sum A_j \phi_j e^{-E_j t/\hbar}$ $\{i = 1\}$
  - The lowest energy solution decay slower

Double well: two mode model

- Assuming two modes, $\Psi = \phi_L \hat{a}_L + \phi_R \hat{a}_R$,with tunneling, J, and on site energy (interactions), U
  - J and U can be measured
- Simple solution
  - Assuming ground (symmetric) and exited (anti-symmetric) states

$$\phi_{L/R} = \frac{1}{\sqrt{2}} \left(\phi_g \pm \phi_e \right)$$

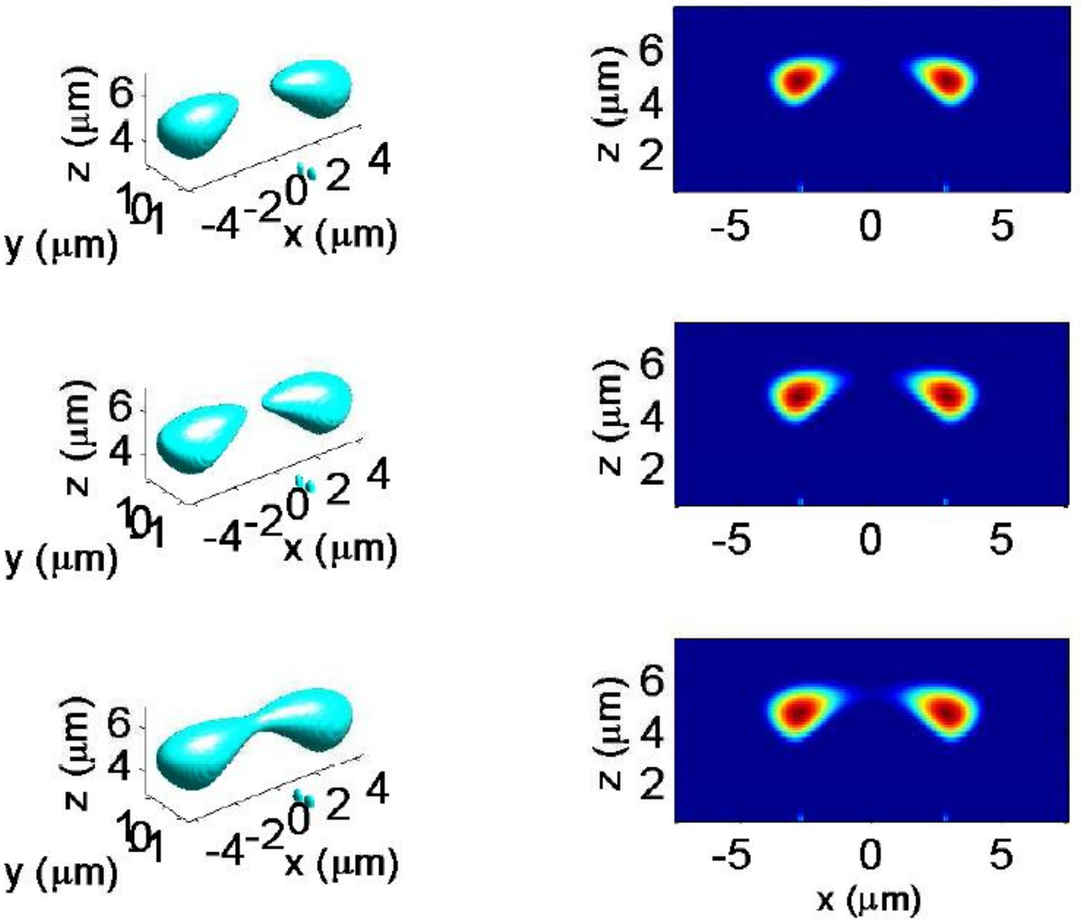
  - Solving GP eq. twice for these two wave functions, and calculating (GP eq. assumes single mode)

$$J = -\int d^3 r \phi_L^* H \phi_R, \; U_{L/R} = g \int d^3 r |\phi_{L/R}|^4$$

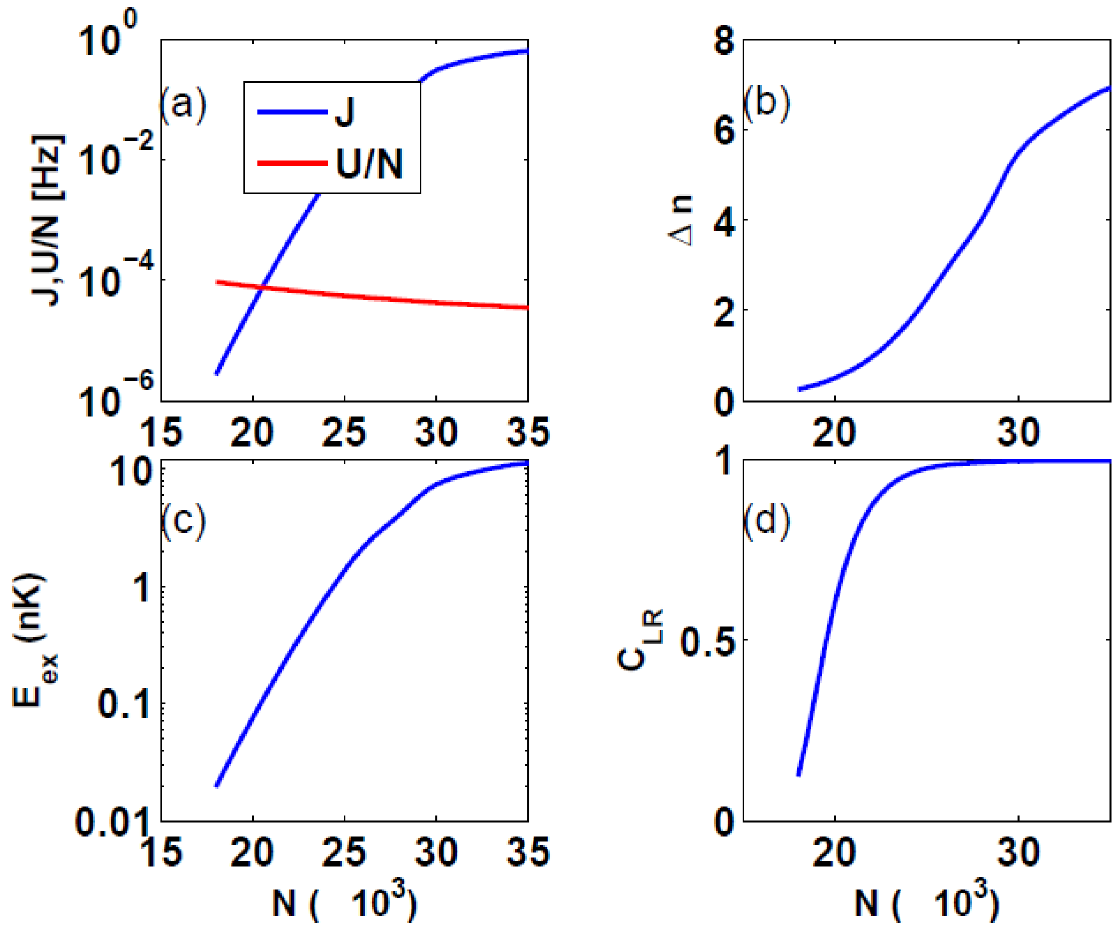
Two mode solution

- We want:
  - Find a solution to: $\phi_{L/R}(\vec{r},t),\ \phi_{g,e}(\vec{r},t)$
  - Find the solution while knowing only the potential
  - Assuming two mode for the entire solution
- Solution
  - we get coupled equations for $\dot{\phi}$ and for $\dot{\hat{a}}$

Initial results: iso-density plots



Initial results: ground state



Summary

- We can solve the equations
- Each point on the graph is about 8hrs of calculations
- We would like to be able to scan the parameters in a more thorough way
  - Different potentials
  - Different atom numbers