Direct numerical testing of stationary shock model with low Mach number shock observations

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1. Introduction

Almost all existing models of collisionless shocks, starting with the one-fluid MHD models and ending with the attempts of kinetic modeling of the shock front, are based on the two assumptions of the shock one-dimensionality and time stationarity. That means that all fields and hydrodynamical variables within the shock front depend only on the single coordinate along the shock normal and do not depend on time, at least at the typical ion timescale. Theoretical and observational descriptions, which use these assumptions, consider only some average profiles of the shock and do not take into account any rippling of the shock surface itself or waves and turbulence inside the shock front. The two assumptions are essential for both theoretical and observational shock physics. The very derivation of the Rankine-Hugoniot relations, connecting the upstream incident plasma parameters to downstream heated plasma parameters, requires one-dimensionality and time stationarity. Theoretical expressions for the field profiles, in particular, for the cross-shock electric field [Scudder, 1995] and for the noncoplanar component of the magnetic field in the front of the quasi-perpendicular shock [Jones and Ellison, 1987, 1991; Gedalin, 1996a], are derived from two fluid hydrodynamics with the necessary implementation of the above assumptions. The description of ion motion and evolution of the ion distribution across the shock, including ion reflection and transmission, as governed by the static electromagnetic fields in the shock front, have been used for the explanation of the magnetic foot and overshoot formation at the supercritical shocks. The estimate of the foot length, which is widely used for the shock velocity determination, also exploits the shock one-dimensionality and stationarity [see, e.g., Skopke et al., 1983, and references therein].

Although this shock stationarity and one-dimensionality is taken for granted by theoreticians for model developments and widely used afterward in the interpretation of experimental data, there is almost no direct analysis (except, probably, by Scudder et al. [1986a, b, c]) of these basic assumptions on the observational data. Nor has it been shown directly that the static fields of the shock front govern the particle behavior, and this interaction results in the buildup of the shock structure.

Numerical simulations [Leroy et al., 1981, 1982; Leroy, 1984] (see also Goodrich [1985] for review) have shown that low Mach number shocks are stationary and one dimensional, with some nonstationarity present in the form of turbulence (waves), the level of which rapidly increases with the increase of the Mach number. These simulations [see also Burgess et al., 1989; Wilkinson, 1991, 1997] elucidated the role of ion reflection/gyration in the plasma thermalization and shock front formation. Kinetic electrons have been further included self-consistently in the full-particle simulations [Quest, 1985; Liewer et al., 1991; Savoini and Lembege, 1994; Krauss-Varban et al., 1995]. Despite high level of sophistication of these hybrid and full-particle simulations, it is still not quite clear what the relative role of static fields and large amplitude waves present in the shock [Brinca et al., 1990; McKean et al., 1995]. Numerical simulations, as well as observations, provide a self-consistent picture with everything (stationary fields and nonstationary waves) included, and separation of different effects is virtually impossible.

The objective of the present paper is to study the consistency of the view that a stationary one-dimensional shock profile can be maintained mainly due to the interaction of charge particles with static fields. There is little or no problem to conclude that the shock is one-dimensional and stationary when the shock is self-consistently simulated [see, e.g., Leroy et al., 1982]. It may be a less easy task for observed shocks [see, e.g., Scudder et al., 1986a]. However, in both cases it is difficult (if possible at all) to conclude unambiguously what is primarily responsible for maintenance of this stationary structure, unless we are able to separate different effects for study. In the present paper we propose a mixed theoretical-numerical-observational approach to the analysis of the underlying hypotheses of the collisionless shock theory. The idea is to take an observed profile of a collisionless shock, properly remove the nonstationary wavy part, the "noise" and then add what is needed on the basis of theoretical knowledge, thus retaining only the stationary fields. The next step is to numerically trace particles (ions) in this structure assuming its time independence and spatial dependence only along the shock normal. This approach allows examination of individual ion trajectories, with particular attention to the possibility of ion reflection. As an additional result, we shall have information about ion heating due to the direct transmission. Finally, we propose to use the pressure balance condition to check the consistency of the numerically determined magnetized plasma state with that measured. This last test should provide a direct check of the basic assumptions. The stationary and one-dimensional shock model requires that total pressure be strictly constant across the shock front. Therefore, if the fields within the shock front were exactly stationary and one dimensional, the magnetic field calculated from pressure balance would coincide with the...
observed magnetic field (see more details in section 4). Substantial deviations of the calculated profile from the measured one would immediately mean that the initial assumptions are not satisfied, and nonstationary (and not one dimensional) fields play a crucial role in the shock structure formation. In the case of reasonable agreement between calculations and observations we would have the estimate of the relative importance of the wave-particle interaction in the shock front. The advantage of the method is that it brings together the observational reality and theoretical predictions using the power of numerical methods, but a successful test requires that both observational errors and theoretical uncertainties be sufficiently small. While this method should be virtually applicable for high Mach number shocks as well, as a first step we restrict ourselves to a low Mach number shock, for which the above conditions seem to be well satisfied.

It should be emphasized that the proposed numerical analysis is not a true simulation and is deliberately carried out as to not be self-consistent. No instabilities are included and noise should be absent. Only in this way are we able to test whether the essential interaction is with the static fields, or the role of waves is equally important.

The paper is organized as follows. In section 2 we describe the necessary preparations of the observational data for the numerical analysis. The principles of the numerical analysis are described briefly in section 3 and the results are presented in section 4. Conclusions and discussion are in section 5.

2. Observational Data and Preparations

For the purposes of the present paper a low Mach number marginally critical (or slightly supercritical) shock crossing measured by ISEE 1 on November 26, 1977, 0610 UT, was selected, for which the determination of the normal coordinates proved to be sufficiently precise [Newbury et al., 1997]. The magnetic field profile for this shock (rotated already into the normal shock coordinates) is shown in Figure 1. It is quite typical for laminar low-Mach number shocks [Mellott and Greenstadt, 1984; Mellott and Livesey, 1987; Farris et al., 1993]. The shock parameters are as follows: the Alfvén Mach number is $M_A = 2.7$, the angle between the shock normal and upstream magnetic field is $\theta = 67^\circ$, ion $\beta_i = 0.16$. The electron temperature was not available and the typical average estimate for solar wind electrons at the Earth orbit [Newbury, 1996] was used to estimate $\beta_e = 0.36$. The shock velocity was calculated to be $V_{sh} = 5.7$ km/s, while the ion inertial length $c/\omega_{pi} = 58$ km [Newbury et al., 1997].

The magnetic field components in the normal incidence frame where the upstream flow velocity is along the shock normal, are found using the coplanarity theorem with the appropriately averaged upstream and downstream magnetic fields. There is no visual foot, only a small overshoot, and the ramp is clearly identified. The noncoplanar component of the magnetic field $B_n$ clearly peaks inside the ramp and becomes relatively small outside it. At the beginning of the ramp the normal component of the magnetic field $B_n$ has an onset of large amplitude fluctuations. The presence of these fluctuations shows that substantial filtering is necessary to remove the time-dependent magnetic fields and retain only that part of the shock structure which may be assumed to be stationary and one dimensional in further numerical analysis. Such filtering has been done by applying wavelet denoising [Chui, 1992; Donoho, 1993] using Daubechies 10 wavelet and removing the 6 finest scales, which corresponds roughly to averaging over 4 s of measurements. The ramp duration is about 10 s so that the safety factor is about 2.5 and the typical ramp scales remain essentially untouched. The denoising parameters are chosen to ensure a monotonic ramp, nearly zero average noncoplanar component ahead of the ramp, and the lowest possible level of fluctuations of the normal component, while retaining at the same time as many small-scale details
3. Numerical Analysis: Description

The objective of the numerical analysis is to trace ions as test particles through the shock structure, determined in section 2, and derive the hydrodynamical characteristics (such as density, hydrodynamical velocity, and pressure) along the shock normal, for the ion distribution which is formed from the initial Maxwellian, to be able eventually to use it for the pressure balance equation:

\[ nm_i V_x^2 + p_{ci,xx} + p_{i,xx} + \frac{B^2}{2\mu_0} = \text{const}, \]  

where for the electrons we continue to use the adiabatic approximation \( p_e \propto n^2 \).

The above means that we analyze the trajectories of ions, whose motion is governed by the static magnetic and electric fields:

\[ m_i \dot{v}_x = e[E_x + (v_y B_z - v_z B_y)], \]
\[ m_i \dot{v}_y = e[E_y + (v_z B_x - v_x B_z)], \]
\[ m_i \dot{v}_z = e(v_x B_y - v_y B_x), \]
where $B$ and $E_x$ are defined in section 2, while the constant motional electric field is $E_y = V_0 B_0 \sin \theta$. The initial ion distribution is taken Maxwellian, with the observed temperature well upstream of the ramp, and ions are counted throughout the shock front using the staying time method [Veltri et al., 1990; Gedalin et al., 1995a, b]. Finding the ion trajectories allows us to determine the ion distribution function $f(v, x)$, which is related to the initial distribution $f(v_0) = f_0(v_0, x_0)$ because of the collisionless nature of motion. Averaging over the distribution (moment derivation) then follows the following prescription [Gedalin and Zilbersher, 1995; Gedalin, 1996b]:

$$
\langle g(v, x) \rangle = \int g(v, x)f(v, x)d^3v
$$

$$
= \int g(v, x)f_0(v_0)|\frac{\partial x}{\partial v_x}|d^3v_0,
$$

where $|\frac{\partial x}{\partial v_x}|$ is the Jacobian of the transformation from $v_0$ at $x_0 = \text{const}$ to $v$ at $x = \text{const}$. The last relation is the direct consequence of the stationarity and one-dimensionality of the field structure.

4. Numerical Analysis: Results

For the statistical analysis we traced 2000 ions across the shock front. Figure 4 shows the trajectories of a subset of 200 ions, taken randomly from the initial Maxwellian far upstream of the shock, together with the magnetic field for convenience. Overall the behavior of ions follows theoretical expectations. Ions are strongly decelerated and slightly deflected at the ramp. Insufficient deceleration by the cross-shock potential causes the downstream ions to retain sufficient energy to maintain strong gyration, which further manifests itself in the collisionless heating, as already clear from Figure 4 and will be shown once again below.

The directly transmitted ion distribution gyrates as a whole, which is consistent with previous numerical simulations [Leroy et al., 1982; Burgess et al., 1989; Wilkinson, 1991, 1997] and theoretical predictions [Gedalin, 1996b]. The collisionless gyrophase mixing, typical for oblique geometry due to the dependence of the ion drift velocity along the shock normal on its peculiar velocity [Gedalin et al., 1996; Gedalin, 1997] is clearly seen. The spatial scale for this mixing is large in the absence of other smoothing factors (as, for example, turbulence).

No truly reflected ions have been observed in the numerical analysis, although some of the ions were almost reflected, making loops inside the ramp. These ions had NIF energies comparable with those that are expected for truly reflected ions, and contributed respectively in the downstream heating. The number of these ions was low (≈ 0.8% for the shock parameters, as given in section 2 and was found to be very sensitive to the incident ion temperature and cross-shock potential. Reduction of the applied cross-shock potential by about 10% reduced the number of these quasi-reflected ions by a factor of 2. An increase of the ion temperature by 12% (with other parameters unchanged) increased the number of quasi-reflected ions by 50%. This phenomenon may be responsible for the discrepancy in two different sets of observations at low-Mach number shocks: in one of which [Thomsen et al., 1985] reflected ions were not found, while in the other [Sckopke et al., 1990] about 2% of reflected ions were reported. In both cases, no magnetic foot was observed, and the reflected ions [Sckopke et al., 1990] were identified by their downstream energies. They were not observed before the ramp. It was argued [Jokipii et al., 1993] that one-dimensionality constrains ion motion to remain within a (generalized) gyroradius of the field line on which the ion started. Lee et al. [1996] have shown that this is automatically satisfied for ion reflection at the shock front and does not impose any additional restrictions even in the one-dimensional case, so that ion reflection is not inhibited.

Figures 5 and 6 present the hydrodynamical variables, obtained by taking the moments of the distribution function found for the traced ions. In Figure 5 we present the ion density (normalized by the incident ion density), three components of the hydrodynamical velocity $V_x$, $V_y$, and $V_z$ (normalized with the upstream plasma velocity, $V_z$ shown by the dashed line), and three components of temperature along with the average temperature (normalized by the upstream ion temperature). In

\[\text{Figure 4. The magnetic field profile and trajectories of 200 ions in the shock front.}\]
Figure 5. Normalized density $n/n_u$, hydrodynamical velocity $V_i/V_u$ (dashed line, $V_z$), and temperature $T_{ii}/T_u$, $T = (1/3) \sum_i T_{ii}$.

Figure 6. Normalized pressure $p_{ij}/n_u m_i V_u^2$. 
Figure 6 we present the full tensor of ion pressure \( p_{ij} \), normalized by \( n_0 m_i V_i^2 \). It is seen that all hydrodynamical variables strongly oscillate starting just behind the ramp. These oscillations are due to the gyration of the directly transmitted ion distribution as a whole [Leroy et al., 1982; Goodrich, 1985; Wilkinson, 1991; Gedalin, 1996b; Wilkinson, 1997] and persist well beyond the shock transition layer, despite the clear smoothing and damping due to collisionless gyrophase mixing. The spatial scale of the oscillations is approximately \( 0.9 V_0 / \Omega_0 \), which coincides with the distance between two neighboring loops of the directly transmitted ion trajectories. The hydrodynamical variables gradually approach their supposedly constant downstream values, although this approach is slow.

As expected, the parallel heating is much weaker than the perpendicular heating. The average temperature, defined as \( T_y = (1/3)(T_{xx} + T_{yy} + T_{zz}) \), also gradually increases and approaches its asymptotic value, although the spatial scale for this approach is even larger than for the density, because of the greater influence of strongly gyrating ions with relatively high energies.

The pressure is highly anisotropic and non-gyrotropic. The diagonal pressure components behave similarly to the temperature components. It is of interest to mention the substantial off-diagonal component \( p_{xy} \) (the two others, \( p_{xz} \) and \( p_{yz} \) are significantly smaller) which builds up within the ramp and persists well beyond it into the downstream region. The appearance of this component is the direct manifestation of the gyration of the downstream ion distribution and can be used for observational verification. The component \( p_{xy} \) enters the expression for the electric field \( E_x \) (see (1)). It is seen that it enhances the (negative) electric field in the upstream half of the ramp, and weakens it in the downstream half of the ramp. Quantitatively the contribution of this term is small (compared to other terms) and its neglect was justified. Further downstream \( p_{xy} \) may dominate in (1), although the corresponding induced electric field remains small.

Our ultimate objective is the test of the consistency of the stationary shock model. This is done by inspecting the corresponding Rankine-Hugoniot relations (RH). We are interested not only in whether the RH equations are satisfied asymptotically, for far upstream and far downstream regions [cf. Scudder et al., 1986a]. Consistency of the shock model requires that these relations be satisfied throughout the shock profile continuously. Of course, removal of possibly time-dependent and three-dimensional noise breaks self-consistency, so we do expect that the RH equations would not be satisfied exactly in our numerical analysis. The degree of disagreement should be used to estimate the importance of the removed fluctuations, so that proper comparison should be done.

More specifically, we assume that the filtered magnetic field is time stationary and depends only in the coordinate along the shock normal. We also adopt the approximation (1) for the cross-shock electric field which is also assumed stationary and one-dimensional. In these conditions the ion dynamics and (collisionless) ion distribution are completely determined by the equations of motion (3)–(5). Yet these equations of motion do not contain any information about back influence of charged particles (currents) on the field structure. In order to estimate the feedback one has to include the resulting currents within the appropriate description of the fields in the shock from, which in our case is (according to the underlying assumptions) the stationary and one-dimensional limit of the two-fluid hydrodynamics. The proper way to do that is to calculate the magnetic field which is generated due to the currents, produced by the ions moving in the initial field. Since the observed magnetic field contains also the time-dependent part, which was filtered out before the numerical analysis of the ion motion, and because of the approximate nature of (1), the system is not made self-consistently, and we expect certain disagreement between the observed and calculated magnetic fields. The amount of this disagreement should provide us with the measure of importance of the time dependent features and three-dimensionality of the shock front. It is worth mentioning that the mere fact that the ion motion and distribution are calculated with fields that are assumed stationary and one-dimensional initially, does not guarantee at all that the calculated magnetic field found from hydrodynamics with this ion distribution be even close to the observed one. Satisfactory agreement requires that the deviations from stationarity and one dimensionality be relatively unimportant.

With this in mind and having restricted ourselves with the second moments of ion distribution, we are left with the momentum conservation, which gives the pressure balance equation which is the \( x \) component of the momentum flux RH, and two perpendicular components, which read (in the same approximation as above and after normalization):

\[
\frac{V_x}{V_u} + \frac{p_{i,xx} + p_{e,xx}}{n_0 m_i V_u^2} + \frac{1}{2 M^2} \left( \frac{||B||}{B_u} \right)^2 = 1 + \frac{1 + \beta_e + \beta_i}{2 M^2},
\]

\[
\frac{V_k}{V_u} + \frac{p_{i,xk}}{n_0 m_i V_u^2} - \frac{1}{M^2} \frac{B_x B_k}{B_u^2} = 0,
\]

where \( k = y, z \), and \( M \) is the Alfvénic Mach number. It is easily seen that in (8) small absolute errors would produce large relative errors, making comparison nonsensible, so that reasonable analysis of the RH conditions can only be done using the pressure balance equation (7), exactly as it is done observationally [Scudder et al., 1986a].

With all above, we present in Figure 7 the comparison between the observed magnetic field with the absolute value of the magnetic field derived from (7) (upon substitution of the numerically calculated hydrodynamical variables with adiabatic
The two profiles agree throughout the whole shock profile, with better agreement well beyond the ramp, where the ion distribution is already noticeably smeared out, than in the ramp vicinity. The difference being small, if can be relatively easily removed by that time-dependent or non-one-dimensional part of the shock which has not be included in the analysis. Wave-particle interactions are responsible for ion scattering. Although this scattering is slow compared to the ramp crossing time, it may be quite sufficient to smooth a little the ion distribution so that the amplitude of the oscillations of hydrodynamical variables be by 10\% lower. Another important mechanism of additional smoothing is related to the three-dimensional shock structure (see, e.g., Scudder et al. [1986a]; A. Mangeney, private communication, 1996): since gyrating ions make substantial excursions along the shock front, we will observe ions entering shock at different points with different conditions and meeting at the same place beyond the ramp [Gedalin, 1997]. Three-dimensional structure of the shock may be also responsible for the enhanced downstream isotropization [Thomas, 1989], although it was observationally shown [Thomsen et al., 1985; Sckopke et al., 1990] that the ion anisotropy and nongyrotropy persist well into downstream in many cases.

To conclude this discussion, the above comparison shows that for low Mach number shocks the interaction between ions and the stationary and one-dimensional structure of the shock front is primarily responsible for the buildup of the shock profile. The remaining work, which has to be done by waves or deviations from one dimensionality, is relatively small.

5. Conclusions

We have studied the applicability of the one-dimensional and stationary model to a real shock by verifying the consistency of our underlying assumptions about the shock structure, with the help of analysis of particle motion and establishing the relation of particle features with the field profile. In doing so we have been successful in the separation of nonstationary (waves, turbulence) and three-dimensional features from the supposedly stationary one-dimensional structure. We have analyzed the ion motion and found that a number of quasi-reflected ions may play the role of truly reflected ions in the formation of high-energy gyrophase-bunched tail in the downstream ion distribution. The downstream distribution of directly transmitted ions gyrates as a whole, which results in strong oscillations of hydrodynamical variables, persisting well beyond the shock transition layer in the downstream region. In reality these oscillations may damp substantially faster as a result of other factors, deliberately not included in this analysis such as: weak nonstationarity of the shock front, rippling of the shock surface (deviations from one dimensionality), and waves and turbulence in the shock. These smoothing factors do not significantly affect the energetics of the ions, which is determined by their interaction with the static fields of the shock front. Finally, we analyzed to what extent the basic assumptions of shock stationarity and one dimensionality are valid. In doing so we assumed that the observed shock profile is stationary and one dimensional. With this assumption theory and numerical analysis allowed us to obtain unambiguous and firm quantitative predictions about ion behavior in the shock front, and to derive macrocharacteristics of the ion distribution. Since the stationary and one dimensional model requires exact maintenance of the pressure balance (constancy of momentum flux) across the shock front, we calculated the magnetic field which is consistent with the model. By demonstrating the agreement between this calculated magnetic field (from pressure balance condition) and the observed magnetic field, we established the consistency of the assumptions of shock one dimensionality and stationarity, having shown also that the shock fine structure is due to the charged particle interaction with the stationary fields, while nonstationary effects may provide fine tuning.

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